Communications
in
Mathematical
Research
32(3)(2016), 241–248

A Class of Ruin Probability Model with Dependent Structure

Wang De-hui, Gao Jia-xing, Xu Zi-li,
Xu Jin-jing and Zhang Xu-li*
(*School of Mathematics, Jilin University, Changchun, 130012*)

Abstract: In this paper, we study a class of ruin problems, in which premiums and claims are dependent. Under the assumption that premium income is a stochastic process, we raise the model that premiums and claims are dependent, give its numerical characteristics and the ruin probability of the individual risk model in the surplus process. In addition, we promote the number of insurance policies to a Poisson process with parameter λ, using martingale methods to obtain the upper bound of the ultimate ruin probability.

Key words: ruin probability, dependent structure, individual risk model, Poisson process

2010 MR subject classification: 91B30

Document code: A

Article ID: 1674-5647(2016)03-0241-08

DOI: 10.13447/j.1674-5647.2016.03.06

1 Introduction

The financial risk theory is a common research direction in mathematics, and theories related to ruin probability are the core contents. More and more researchers are getting interested in the theoretical foundation of ruin theory; moreover, they apply them to all aspects of the social economy. In reality, every insurance company must consider the corresponding ruin probability when radicating pricing strategies. Therefore, it is important to study the relationships between premium and claim income. Lundberg[1] and Cramer[2] put forward and perfected the well-known classical ruin model. However, the classical compound-Poisson risk model $U(t) = u + ct - \sum\limits_{i=1}^{N(t)} Y_i$ is ideal, for it requires that the point process $\{N(t),\ t \geq 0\}$ is a Poisson process, and the cost of claims $\{Y_i,\ i \in \mathbf{N}^+\}$ is a sequence of independent and identically distributed variables with finite mean, independent of $\{N(t),\ t \geq 0\}$ and the

Received date: May 11, 2015.
Foundation item: The NSF (11271155, J1310022) of China.
* **Corresponding author**.
E-mail address: wangdh@jlu.edu.cn (Wang D H), zhangxl@jlu.edu.cn (Zhang X L).

rate of premiums income per unit time is a constant (see [3]). As a result, some scholars made improvements, and they think the point process of insurance policies should not be a Poisson process. For example, Mao[4] introduced a generalized Poisson-geometric process with two parameters of claim numbers. Gerber[5] introduced a mixed Poisson risk process. Shiu[6] established a discrete-time model, where the number of claims process is assumed to be binomial. Grandell[7] discussed the ruin probability when the point process is a common or balanced renewal process. In the case of heavy-tailed claims, Kluppelberg and Mikosch[8] gave large deviations of ruin probability when the point process is a renewal process. Moreover, there exist dependence between claims, between interclaim times, and between claims and interclaim times. Asmussen and Albrecher[9] pointed out a causal dependence may be needed in practice, where for instance the size of a claim determines the distribution of the next interclaim times. When both claim sizes and interarrival times follows a certain dependent structure, Wang *et al.*[10] calculated the expression for the finite time ruin probability of a nonstandard risk model with a constant interest rate. Ambagaspitiya[11] also talked about the dependent structure between claim sizes and claim occurrence times. Albrecher and Boxma[12] considered the situation that the distribution of the time between two claim occurrences depends on the previous claim size. Zhang[13] proposed a dependent model between different insurance categories. And yet for all that, all of them ignored the dependent structure between premiums and claims. In order to be precise and compensate for financial mathematics theory (see [14]–[15]), in this paper, we raise a ruin model where premiums and claims are dependent based on the premium income being a stochastic process

$$U_n = u + \sum_{i=1}^{n} X_i - \sum_{i=1}^{n} h(X_i) I_i. \tag{1.1}$$

We focus on calculating the numerical characteristics of the surplus process when the claim income is a monotonous increasing function of the premium income, and use martingale methods to obtain the upper bound of the probability of the ultimate ruin. We begin by a specific example of Ping An Property and Casualty Insurance Company of China and then raise our model, giving its numerical characteristics and calculating its ruin probability approximately. Finally, we generalize the number of insurance policies to a Poisson counting process with parameter λ and give its upper bound of the ruin probability.

2 The Ruin Model with Dependent Structure Proposed

According to annual integrated accident insurance, which was promoted by Ping An Property and Casualty Insurance Company of China in 2014, claims of accidental death can be categorized into the following sorts (see Table 2.1).

Table 2.1 Relationship between claims and insured amounts

Claims (10 000 yuan)	5	10	20	30	40	50
Insured amount (yuan)	46.5	93	186	279	372	456