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Two Optimal Inequalities Related to the Sándor-Yang Type Mean and One-parameter Mean

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Abstract: In this paper, we establish two optimal the double inequalities for Sándor-Yang type mean and one-parameter mean.

Key words: Sándor-Yang type mean, p-th one-parameter mean, Neuman-Sándor mean, second Seiffert mean, inequality

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1 Introduction

Let p, $q \in \mathbf{R}$ and a, $b > 0$ with $a \neq b$. The Stolarsky means $S_{p,q}(a,\ b)$ were defined by Stolarsky[1] as

$$S_{p,q}(a,\ b) = \begin{cases} \left(\dfrac{q(a^p - b^p)}{p(a^q - b^q)}\right)^{\frac{1}{p-q}} & \text{if } p \neq q,\ pq \neq 0; \\ \left(\dfrac{a^p - b^p}{p(\ln a - \ln b)}\right)^{\frac{1}{p}} & \text{if } p \neq 0,\ q = 0; \\ \left(\dfrac{a^q - b^q}{q(\ln a - \ln b)}\right)^{\frac{1}{q}} & \text{if } p = 0,\ q \neq 0; \\ \exp\left\{\dfrac{a^p \ln a - b^p \ln b}{a^p - b^p} - \dfrac{1}{p}\right\} & \text{if } p = q \neq 0; \\ \sqrt{ab} & \text{if } p = q = 0. \end{cases} \tag{1.1}$$

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It is well known that the Stolarsky means $S_{p,q}(a,\ b)$ are continuous and strictly increasing with respect to $p, q \in \mathbf{R}$ and $a, b > 0$, and include many famous means, for example, $S_{1,0}(a,\ b) = L(a,\ b)$ is the logarithmic mean, $S_{1,1}(a,\ b) = I(a,\ b)$ is the identric (exponential) mean, $S_{2,1}(a,\ b) = A(a,\ b)$ is the arithmetic mean, $S_{\frac{3}{2},\frac{1}{2}}(a,\ b) = He(a,\ b)$ is the Heronian mean,

$$S_{2p,p}(a,\ b) := M_p(a,\ b) = \begin{cases} \left(\dfrac{a^p + b^p}{2}\right)^{\frac{1}{p}} & \text{if } p \neq 0; \\ \sqrt{ab} & \text{if } p = 0 \end{cases}$$

is the p-th power mean of a and b, while

$$S_{p+1,p}(a,b) := J_p(a,\ b) = \begin{cases} \dfrac{p}{p+1} \cdot \dfrac{a^{p+1} - b^{p+1}}{a^p - b^p} & \text{if } p(p+1) \neq 0; \\ \dfrac{a-b}{\ln a - \ln b} & \text{if } p = 0; \\ \dfrac{ab(\ln a - \ln b)}{a-b} & \text{if } p = -1 \end{cases}$$

is called the pth one-parameter mean of a and b.

The Schwab-Borchardt mean $SB(a,\ b)$ of two positive real numbers a and b is defined by

$$SB(a,\ b) = \begin{cases} \dfrac{\sqrt{b^2 - a^2}}{\arccos\left(\dfrac{a}{b}\right)} & \text{if } a < b; \\ \dfrac{\sqrt{a^2 - b^2}}{\cosh^{-1}\left(\dfrac{a}{b}\right)} & \text{if } a > b \end{cases}$$

(see [2]–[4]). It is known that the Schwab-Borchardt mean $SB(a,\ b)$ is also strictly increasing in both a and b, nonsymmetric and homogeneous of degree one with respect to a and b. Many symmetric bivariate means values are the special cases of the Schwab-Borchardt mean. For instance,

$$P(a,\ b) = \frac{a-b}{2\arcsin\dfrac{a-b}{a+b}} = SB[G(a,\ b),\ A(a,\ b)]$$

is the first Seiffert mean,

$$T(a,\ b) = \frac{a-b}{2\arctan\dfrac{a-b}{a+b}} = SB[A(a,\ b),\ Q(a,\ b)]$$

is the second Seiffert mean,

$$NS(a,\ b) = \frac{a-b}{2\sinh^{-1}\dfrac{a-b}{a+b}} = SB[Q(a,\ b),\ A(a,\ b)]$$

is the Neuman-Sándor mean, and the logarithmic mean $L(a,\ b)$ can be rewritten as

$$L(a,\ b) = \frac{a-b}{2\tanh^{-1}\dfrac{a-b}{a+b}} = SB[A(a,\ b),\ G(a,\ b)],$$

where $Q(a,\ b) = \sqrt{\dfrac{a^2+b^2}{2}}$ is the quadratic mean. Then it is easy to see that the inequalities