Communications
in
Mathematical
Research
31(2)(2015), 108–118

Statistical Inference for the Parameter of Rayleigh Distribution Based on Progressively Type-I Interval Censored Sample

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Abstract: In this paper, the estimation of parameters based on a progressively type-I interval censored sample from a Rayleigh distribution is studied. Different methods of estimation are discussed. They include mid-point approximation estimator, the maximum likelihood estimator, moment estimator, Bayes estimator, sampling adjustment moment estimator, sampling adjustment maximum likelihood estimator and estimator based on percentile. The estimation procedures are discussed in details and compared via Monte Carlo simulations in terms of their biases.

Key words: EM algorithm, maximum likelihood estimation, moment method, Bayes estimation, Rayleigh distribution

2010 MR subject classification: 62N01, 62F10

Document code: A

Article ID: 1674-5647(2015)02-0108-11

DOI: 10.13447/j.1674-5647.2015.02.02

1 Introduction

The Rayleigh distribution provides a population model which is useful in many areas of statistics, including life-testing, reliability and operations research. The probability density function and the cumulative distribution function of the Rayleigh distribution are given by

$$f(x;\alpha) = 2\alpha x \exp\{-\alpha x^2\}, \qquad 0 < x < \infty,$$

and

$$F(x;\alpha) = 1 - \exp\{-\alpha x^2\}, \qquad 0 < x < \infty,$$

respectively, where $\alpha > 0$ is the parameter.

Received date: March 9, 2013.

Foundation item: The NSF (11271155, 11001105, 11071126, 10926156, 11071269) of China, the Specialized Research Fund (20110061110003, 20090061120037) for the Doctoral Program of Higher Education, the Scientific Research Fund (201100011, 200903278) of Jilin University, and the NSF (20101596, 20130101066JC) of Jilin Province.

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The failure rate of the Rayleigh distribution is an increasing linear function of time. Therefore, when the failure times are distributed according to the Rayleigh law, an intense aging of the equipment takes place. This means that as time increases the reliability function decreases at a much higher rate than in the case of exponential distribution (see [1]).

Inferences for the Rayleigh distribution were discussed by several authors. Based on type II censored data, Harter and Moore[2] derived an explicit form for the maximum likelihood estimator of the unknown parameter α, and Howlader and Hossain[3] considered Bayesian estimation and prediction from Rayleigh. Moreover, Dyer and Whisenand[4–5] obtained the best linear unbiased estimator of α based on complete sample, censored sample and selected order statistics. Doubly censored samples were also considered (see [6–8]). Recently, Wu *et al.*[9] considered the Bayesian estimator and prediction intervals for future observations based on progressively type II censored samples. Solimana and Al-Aboud[10] discussed Bayesian and non-Bayesian estimators of the parameter, and some lifetime parameters such as the reliability and hazard functions based on a set of upper record values from a Rayleigh distribution.

In many life-testing and reliability studies, experimental units can be removed progressively from the experiment during the different stages of the testing. The major reasons for removal are saving the working experimental units for future use or lowering the cost. Data obtained from such experiments are called progressive censored data. Aggarwala[11] explored a union of type-I interval and progressive censoring and developed the statistical inference for exponential distribution for progressive type-I interval censored sample. Since then, statistical analysis for progressive type-I interval censored data has been studied by many authors, for example, Chen and Lio[12], Lin *et al.*[13], Yan *et al.*[14] and Ng and Wang[15].

Progressive type-I interval censoring scheme can be described as follows. Consider n units which put on a test at time $t_0 = 0$. Units are inspected at m precified times t_1, t_2, $\cdots$, t_m, where t_m is the scheduled time to terminate the experiment. At the ith inspection time t_i, the number, X_i, of failures within $(t_{i-1},\ t_i]$ is recorded and R_i surviving items are randomly removed from the life testing for $i = 1, 2, \cdots, m$. As the numbers of surviving units at time t_1, t_2, $\cdots$, t_m are random variables, the numbers of removal R_1, $\cdots$, R_m can be determinated as pre-specified percentages of the remaining surviving units. For example, given pre-specified percentage values $p_1, \cdots, p_{m-1}$ and $p_m = 1$, for withdrawing at $t_1 < t_2 < \cdots < t_m$, respectively, $R_i = [p_i y_i]$ at each inspection time t_i, where $[p_i y_i]$ denotes the largest integer which is smaller than or equal to $p_i y_i$. In this article, we discuss the estimation of parameters based on a progressively type-I interval censored sample from the Rayleigh distribution. In addition, using the mid-point approximation estimate, maximum likelihood estimate and moment estimate methods suggested by Ng and Wang[15], we further develop several new estimate procedures, such as Bayes estimate, sampling adjustment moment estimate and maximum likelihood estimate and estimate based on percentile.

The rest of this article is organized as follows. In Section 2, we describe the specific estimate procedure of the above estimate methods. Simulation results are provided in Section 3.