Two Dimensional Indecomposable Modules over Infinite Dimensional Hereditary Path Algebras

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Abstract: Non-isomorphic two dimensional indecomposable modules over infinite
dimensional hereditary path algebras are described. We infer that none of them can
be determined by their dimension vectors.

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1   Introduction

Let k be an algebraically closed field, k^* be the set of non-zero elements of k. Given a
k-algebra A over k, by an A-module we mean a finite dimensional left A-module.

It is known that any finite dimensional basic k-algebra is isomorphic to the corresponding
quiver path algebra modulo the associated admissible ideal, see [1–3]. Modules over finite
dimensional basic k-algebras can be described by representations of the associated quiver
where a group of linear transformations, namely a group of matrices satisfying some conditions are involved. These methods also fit for finite dimensional module categories over
infinite dimensional algebras.

Given an algebra A over k, whether it is finite or infinite dimensional, we know that a main object studying the module category of A is to determine all pairwise non-isomorphic indecomposable modules over A.

In the literature, there is not much information on module categories over infinite dimensional path algebras. So as a try step, we want to study module categories over infinite dimensional hereditary path algebras, namely infinite quiver path algebras without relations. And as a first step we want to describe all pairwise non-isomorphic two dimensional indecomposable modules over these algebras. The main aim of this paper is to achieve this work.

Since the task is to classify two dimensional indecomposable modules over infinite dimensional path algebras, we need only consider infinite dimensional path algebras which are support algebras of two dimensional indecomposable modules.

Firstly, we present all pairwise non-isomorphic infinite dimensional hereditary path algebras which are support algebras of two dimensional indecomposable modules, in Proposition 1.1.

Proposition 1.1 *The quivers which provide infinite dimensional hereditary path algebras which are support algebras of two dimensional indecomposable modules are completely presented as follows*:

(1)

(2)

(3)

(4)

(5)

(6)