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Stationary Solutions for a Generalized Kadomtsev-Petviashvili Equation in Bounded Domain

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Abstract: In this work, we are mainly concerned with the existence of stationary solutions for a generalized Kadomtsev-Petviashvili equation in bounded domain of $\mathbf{R}^n$. We utilize variational method and critical point theory to establish our main results.

Key words: generalized Kadomtsev-Petviashvili equation, stationary solution, critical point theory, variational method
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1 Introduction

We investigate the stationary solutions for the generalized Kadomtsev-Petviashvili equation in bounded domain of $\mathbf{R}^n$ as follows:

$$\begin{cases} \dfrac{\partial^3}{\partial x^3}u(x, y) + \dfrac{\partial}{\partial x}[f(u(x, y)) + \lambda u(x, y)] = D_x^{-1}\Delta_y u(x, y) \qquad \text{in } \Omega, \\ D_x^{-1}u|_{\partial\Omega} = 0, \quad u|_{\partial\Omega} = 0, \end{cases} \tag{1.1}$$

where λ is a parameter, $\Omega \subset \mathbf{R}^n$ is a bounded domain with smooth boundary,

$$D_x^{-1}h(x, y) := \int_{-\infty}^{x} h(s, y)\mathrm{d}s$$

denotes the inverse operator,

$$(x, y) := (x, y_1, \cdots, y_{n-1}) \in \mathbf{R} \times \mathbf{R}^{n-1}, \qquad n \geq 2,$$

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and

$$\Delta_y := \frac{\partial^2}{\partial y_1^2} + \frac{\partial^2}{\partial y_2^2} + \cdots + \frac{\partial^2}{\partial y_{n-1}^2}.$$

In this paper, we utilize variational methods and some critical point theorems to study the stationary solutions for the generalized Kadomtsev-Petviashvili equation (1.1).

In mathematics and physics, the Kadomtsev-Petviashvili equation (KP equation), named after Boris Borisovich Kadomtsev and Vladimir Iosifovich Petviashvili, is a partial differential equation to describe nonlinear wave motion. To the best of our knowledge, by virtue of variational method and critical point theory, the recent papers on the subject can be found in [1–7] and the references therein. Generally, it reads

$$\frac{\partial}{\partial t} w(t, x, y) + \frac{\partial^3}{\partial x^3} w(t, x, y) + \frac{\partial}{\partial x} f(w(t, x, y)) = D_x^{-1} \Delta_y w(t, x, y), \tag{1.2}$$

where

$$(t, x, y) := (t, x, y_1, \cdots, y_{n-1}) \in \mathbf{R}^+ \times \mathbf{R} \times \mathbf{R}^{n-1}, \qquad n \geq 2,$$

D_x^{-1} and Δ_y are as in (1.1). A solitary wave is a solution of the form

$$w(t, x, y) = u(x - ct, y),$$

where $c > 0$ is fixed. Liang and Su[1] considered the case that the non-constant weight function for generalized Kadomtsev-Petviashvili equation, and Xuan[2] dealt with the case, where $N \geq 2$ and $f(u)$ satisfies some superlinear conditions. Their main tool in [1–2] is the famous Ambrosetti-Rabinowitz mountain pass theorem. Wang and Willem[3] built multiplicity results of solitary waves of (1.2) by Lyusternik-Schnirelman theory. He and Zou[4] established the existence of nontrivial solitary waves for (1.1) (the case $\lambda = 0$) by Szulkin and Zou's linking theorem. More precisely, in [4], $f(u)$ is required to satisfy the following conditions:

(i) $f \in C(\mathbf{R}, \mathbf{R})$, $f(0) = 0$ and for some p with $2 < p < \bar{p} = \dfrac{2(2N-1)}{2N-3}$, $d > 0$, there holds

$$|f(u)| \leq d(1 + |u|^{p-1}),$$

moreover,

$$f(t) = o(|t|) \qquad \text{as } t \to 0;$$

(ii) There exists a $v_0 \in Y = \{u_x : u \in C_0^\infty(\mathbf{R}^N)\}$ such that

$$\frac{F(tv_0)}{t^2} \to \infty \qquad \text{as } t \to \infty;$$

(iii) There exist $\mu \in \Big[\dfrac{(2N-1)(p-2)}{2}, p\Big]$ and $c_2 > 0$ such that

$$c_2 |u|^\mu \leq f(u)u - 2F(u), \qquad u \in \mathbf{R};$$

(iv) $uf(u) \geq 0$ for all $u \in \mathbf{R}$.

Under the above conditions, (1.1) possesses a nontrivial solution (see Theorem 1.1 in [4]). Note that the Ambrosetti-Rabinowitz type superlinear condition is crucial in proving the boundedness of the Palais-Smale sequence (see (f_3) of page 16 in [2]). However, in [4],