Communications in Mathematical Research
30(2)(2014), 139–156

Pseudo Almost Automorphic Solutions for Non-autonomous Stochastic Differential Equations with Exponential Dichotomy

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Communicated by Li Yong

Abstract: In this paper, we consider the existence and uniqueness of the solutions which are pseudo almost automorphic in distribution for a class of non-autonomous stochastic differential equations in a Hilbert space. In conclusion, we use the Banach contraction mapping principle and exponential dichotomy property to obtain our main results.

Key words: pseudo almost automorphy, exponential dichotomy, non-autonomous stochastic differential equation
2010 MR subject classification: 60H25, 34C27, 34F05, 34G20
Document code: A
Article ID: 1674-5647(2014)02-0139-18
DOI: 10.13447/j.1674-5647.2014.02.05

1 Introduction

Liu and Sun[1] introduced the concept of almost automorphy in distribution and studied the almost automorphy in distribution solutions of stochastic differential equations driven by Lévy noise. Chen and Lin[2] researched the square-mean pseudo almost automorphic process and its applications.

In this paper, we consider the existence and uniqueness of the solutions which are pseudo almost automorphic in distribution for a class of non-autonomous stochastic differential equations of the form

$$\mathrm{d}x(t) = A(t)x(t)\mathrm{d}t + f(t, x(t))\mathrm{d}t + g(t, x(t))\mathrm{d}W(t), \qquad t \in \mathbf{R}, \tag{1.1}$$

where $A(t)$ is a family of closed linear operators satisfying the Acquistapace-Terrani conditions (see [3–4]), $f(t, x)$, $g(t, x)$ are square-mean pseudo almost automophic in $t \in \mathbf{R}$ for

Received date: Oct. 9, 2012.
Foundation item: The Undergraduate Research Training Program Grant (J1030101) and the NSF (11271151) of China.
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each $x \in \mathcal{L}^2(P, H)$, and f, g are assumed to satisfy Lipschitz conditions with respect to x.

This paper is organized as follows. In Section 2, we provide definitions, lemmas and propositions. In Section 3, we prove our main result.

2 Preliminaries

In this section, we provide some preliminaries. The readers may find more details in [1–9].

2.1 The Norm of the Space

Throughout this paper, we assume that $(H, \|\cdot\|)$ is a real separable Hilbert space. Let $(\Omega, \mathcal{F}, P)$ be a complete probability space. The notation $\mathcal{L}^2(P, H)$ stands for the space of all H-valued random variables x such that

$$E\|x\|^2 = \int_\Omega \|x\|^2 \mathrm{d}P < \infty.$$

For $x \in \mathcal{L}^2(P, H)$, let

$$\|x\|_2 = \Big(\int_\Omega \|x\|^2 \mathrm{d}P\Big)^{\frac{1}{2}}.$$

Then it is routine to check that $\mathcal{L}^2(P, H)$ is a Hilbert space equipped with the norm $\|\cdot\|_2$. Let $W(t)$ be a two-sided standard one-dimensional Brownian motion defined on the filtered probability space $(\Omega, \mathcal{F}, P, \mathcal{F}_t)$, where $\mathcal{F}_t = \sigma\{W(u) - W(v);\, u, v \le t\}$.

2.2 Square-mean Pseudo Almost Automorphic

Definition 2.1[1] *A stochastic process $x : \mathbf{R} \to \mathcal{L}^2(P, H)$ is said to be $\mathcal{L}^2$-continuous if for any $s \in \mathbf{R}$,*

$$\lim_{t\to s} E\|x(t) - x(s)\|^2 = 0.$$

Note that if an H-valued process is $\mathcal{L}^2$-continuous, then it is necessarily stochastically continuous.

Definition 2.2[2] *A stochastic process $x : \mathbf{R} \to \mathcal{L}^2(P, H)$ is said to be $\mathcal{L}^2$-bounded if there exists an $M > 0$ such that*

$$E\|x(t)\|^2 \le M, \qquad t \in \mathbf{R}.$$

The collection of all $\mathcal{L}^2$-bounded continuous processes is denoted by $SBC(\mathbf{R}; \mathcal{L}^2(P, H))$.

Definition 2.3[2] *By a stochastic process $x \in SBC_0(\mathbf{R}; \mathcal{L}^2(P, H))$, we mean that*

$$x \in SBC(\mathbf{R}; \mathcal{L}^2(P, H)) \qquad \text{and} \qquad \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} E\|x(t)\|^2 \mathrm{d}t = 0.$$

Definition 2.4[1] *An $\mathcal{L}^2$-continuous stochastic process $x : \mathbf{R} \to \mathcal{L}^2(P, H)$ is said to be square-mean almost automorphic if every sequence of real numbers $\{s_n'\}$ has a subsequence $\{s_n\}$ such that for some stochastic processes $y : \mathbf{R} \to \mathcal{L}^2(P, H)$,*

$$\lim_{n\to\infty} E\|x(t + s_n) - y(t)\|^2 = 0 \qquad \text{and} \qquad \lim_{n\to\infty} E\|y(t - s_n) - x(t)\|^2 = 0$$