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Existence of Solutions to Generalized Vector Quasi-variational-like Inequalities with Set-valued Mappings

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Abstract: In this paper, we introduce and study a class of generalized vector quasi-variational-like inequality problems, which includes generalized nonlinear vector variational inequality problems, generalized vector variational inequality problems and generalized vector variational-like inequality problems as special cases. We use the maximal element theorem with an escaping sequence to prove the existence results of a solution for generalized vector quasi-variational-like inequalities without any monotonicity conditions in the setting of locally convex topological vector space.

Key words: generalized vector quasi-variational-like inequality, maximal element theorem, upper semicontinuous diagonal convexity, locally convex topological vector space

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1 Introduction

Vector variational inequality was first introduced and studied by Giannessi[1] in the setting of finite-dimensional Euclidean spaces. This is a generalization of a scalar variational inequality to the vector case by virtue of multi-criteria consideration. Throughout the development over the last twenty years, existence theorems of solutions of vector variational inequalities in various situations have been studied by many authors (see, for example, [2–5] and the references therein). Recently, Peng and Rong[6], Ahmad and Irfan[7] and Xiao *et al.*[8]

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proved some existence theorems of solutions to a class of generalized nonlinear variational inequalities.

In this paper, we introduce a new class of generalized vector quasi-variational-like inequality problems and utilize the maximal element theorem with an escaping sequence to prove the existence of its solutions in the setting of locally convex topological vector spaces (locally convex spaces, in short). Some results of [6–8] are improved and extended.

2 Preliminaries

Let Z be a locally convex space and X be a nonempty convex subset of a Hausdorff topological vector space E (t.v.s., in short). We denote by $L(E,Z)$ the space of all continuous linear operators from E into Z and by $\langle l, x\rangle$ the evaluation of $l \in L(E,Z)$ at $x \in E$. Let $L(E,Z)$ be a space equipped with σ-topology. By the corollary of Schaefer (see page 80 in [9]), $L(E,Z)$ becomes a locally convex space. By Ding and Tarafdar[10], the bilinear mapping $\langle \cdot , \cdot \rangle : L(E,Z) \times E \to Z$ is continuous.

Let $\mathrm{int}S$ and $\mathrm{co}S$ denote the interior and convex hull of a set S, respectively, $C : X \to 2^Z$ be a set-valued mapping such that $\mathrm{int}C(x) \neq \emptyset$ for each $x \in X$, and $\eta : X \times X \to E$ be a vector-valued mapping. Let $T : X \to 2^{L(E,Z)}$, $D : X \to 2^X$, $A : L(E,Z) \to 2^{L(E,Z)}$ and $H : X \times X \to 2^Z$ be four set-valued mappings. We consider the following generalized vector quasi-variational-like inequality problem (GVQVLIP, in short):

Find $\bar{x} \in X$ such that $\bar{x} \in D(\bar{x})$ and for all $y \in D(\bar{x})$, there exists $\bar{s} \in T(\bar{x})$ satisfying

$$\langle A\bar{s}, \eta(y,\bar{x})\rangle + H(\bar{x},y) \not\subseteq -\mathrm{int}C(\bar{x}). \tag{2.1}$$

The following problems are special cases of GVQVLIP.

(i) For all $x \in X$, if $D(x) = X$, then (2.1) reduces to

$$\langle A\bar{s},\ \eta(y,\bar{x})\rangle + H(\bar{x},y) \not\subseteq -\mathrm{int}C(\bar{x}), \qquad y \in X, \tag{2.2}$$

which has been studied by Xiao *et al.*[8]

Find $\bar{x} \in X$, such that there exists $\bar{s} \in T(\bar{x})$ satisfying (2.2).

(ii) Let $A = I$ be a single-valued mapping and $H \equiv 0$. Then (2.1) reduces to

$$\langle \bar{v},\ \eta(y,\bar{x})\rangle \notin -\mathrm{int}C(\bar{x}), \tag{2.3}$$

which has been studied by Peng and Rong[6].

Find $\bar{x} \in X$, such that $\bar{x} \in D(\bar{x})$ and for all $y \in D(\bar{x})$, there exists $\bar{v} \in T(\bar{x})$ satisfying (2.3).

For suitable and appropriate choice of the mappings D, T, A, H, η, one can obtain various new and previously known variational inequality problems as special cases (see [6], [8] and the references therein).

In order to prove the main results, we need the following definitions and lemmas.

Let X be a topological space. A subset S of X is said to be compactly open (respectively, compactly closed) in X if for any nonempty compact subset K of X, $S \bigcap K$ is open (respectively, closed) in S. Let Y be a topological space and $T : X \to 2^Y$ be a set-valued mapping. Then, T is said to be open valued if the set $T(x)$ is open in X for each $x \in X$. T is said to