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An Evolving Random Network and Its Asymptotic Structure

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Abstract: In this paper, we propose an evolving random network. The model is a linear combination of preferential attachment model and uniform model. We show that scaling limit distribution of the number of leaves at time n is approximated by nomal distribution and the proportional degree sequence obeys power law. The branching structure and maximum degree are also discussed in this paper.
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1 Introduction

In the recent ten years, there has been much interest in understanding the properties of real large-scale complex networks which describe a wide range of systems in nature and society such as the internet, the world wide web, protein interaction networks, brain cell networks, science collaboration graph, web of human sexual contact, phone-call networks, power and neural networks, etc. (see [1]). In pursuit of such understanding, mathematicians, biologists and physicists usually used random graphs to model all these real-life networks. For a general introduction, we can see [2–7].

In the model of [2], when $m = 1$, the resulting graph is a tree. These scale-free trees have known since the 1980s as nonuniform random recursive tree. Two nearly identical classes of these trees are random recursive trees with attraction of vertices proportional to the degrees and random plane-orient recursive tree (see [8–13]). Later, the model was generalized by Bollobás *et al.*[14] and Mahmoud[15], in which the probability of choosing an old vertex is $(k + \beta)/S_n$, instead of $k/2n$, with a given $\beta > -1$, where k is the degree of the vertex and

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$S_n = (2+\beta)n + \beta$ is the sum of all weights of vertices.

In this paper, we propose a kind of evolving random networks which shows tree structure. We start with two vertices connecting by a single edge, and at every time step, we add a vertex and connect it with one of the existing vertices according to the following rule:

I. with probability $1-\alpha$ we chose an existing vertex with equal probability;

II. with probability α, we choose an existing vertex with the probability proportional to the degree, that is k/S_n, where k is the degree of the vertex chosen and $S_n = 2n$ is the total degree of vertices.

The model shows many different properties from the existing model, and we show those in following sections.

2 Normal Distribution of Sacled Number of Leaves

In this section and herein after, $D_k(n)$ denotes the number of vertices with degree k at time n, and

$$E_s D_k(n) = E[D_k(n)(D_k(n)-1)(D_k(n)-2)\cdots(D_k(n)-s+1)]$$

denotes the s-th factorial moment of $D_k(n)$. When $k=1$, we know that it is the number of leaves in the tree structure, and we have the following lemma:

Lemma 2.1 *For $k=1$, we have*

$$ED_1(n) = \frac{2}{4-\alpha}n + o(n) \tag{2.1}$$

and

$$E_2 D_1(n) = \frac{4n^2}{(4-\alpha)^2} + o(n^2),$$

where n denotes the evolving time.

Proof. Noticing the evolution of the random network, we can see that for $n>1$, the number of vertices of degree 1 either remains unchange if we attach v_{n+1} to a vertex with degree 1 or increases by 1 if we attach v_{n+1} to vertices of degree larger than 1. Hence,

$$E(D_1(n+1)|D_1(n)) = D_1(n)\Big(1-\frac{2-\alpha}{2n}\Big)+1.$$

Taking expectation of both sides, we can write

$$ED_1(n+1) = \Big(1-\frac{2-\alpha}{2n}\Big)ED_1(n)+1.$$

The solution of this recurrence problem with the boundary condition $ED_1(0)=0$ is given by

$$ED_1(n+1) = \sum_{j=1}^{n}\prod_{i=j}^{n}\frac{j-1+\dfrac{\alpha}{2}}{j} = \sum_{j=1}^{n}\frac{\Gamma\Big(n+\dfrac{\alpha}{2}-1\Big)\Gamma(j-1)}{\Gamma\Big(j+\dfrac{\alpha}{2}-2\Big)\Gamma(n)}+1.$$

By the Stirling's formula, we obtain

$$ED_1(n+1) = \sum_{j=1}^{n}\Big(\frac{j}{n}\Big)^{1-\frac{\alpha}{2}} + o(n).$$