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Some Characterizations of Strongly Semisimple Ordered Semigroups

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Abstract: In this paper, the concept of quasi-prime fuzzy left ideals of an ordered semigroup S is introduced. Some characterizations of strongly semisimple ordered semigroups are given by quasi-prime fuzzy left ideals of S. In particular, we prove that S is strongly semisimple if and only if each fuzzy left ideal of S is the intersection of all quasi-prime fuzzy left ideals of S containing it.

Key words: ordered semigroup, ordered fuzzy point, quasi-prime fuzzy left ideal, strongly semisimple ordered semigroup

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1 Introduction

The fundamental concept of a fuzzy set, introduced by Zadeh[1] in 1965, provides a natural framework for generalizing several basic notions of algebra. Following the terminology given by Zadeh[1], Kehayopulu and Tsingelis[2] first considered fuzzy sets in ordered semigroups, and defined "fuzzy" analogues for several notations, which have proven to be useful in the theory of ordered semigroups. Moreover, they proved that each ordered groupoid can be embedded into an ordered groupoid having the greatest element (poe-groupoid) in terms of fuzzy sets (see [3]). A theory of fuzzy sets on ordered semigroups has been recently developed (see [4–9]). The concept of ordered fuzzy points of an ordered semigroup S was first introduced by Xie and Tang[7], and prime fuzzy ideals of an ordered semigroup

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S were studied in [8]. Authors also introduced the concepts of weakly prime fuzzy ideals, completely prime fuzzy ideals, completely semiprime fuzzy ideals and weakly completely prime fuzzy ideals of an ordered semigroup S, and established the relations among the five types of ideals. Furthermore, Xie and Tang[7] has characterized weakly prime fuzzy ideals, completely semiprime fuzzy ideals and weakly completely prime fuzzy ideals of S by their level ideals.

As we know, fuzzy ideals (left, right ideals) with special properties of ordered semigroups always play an important role in the study of ordered semigroups structure. The ordered fuzzy points of an ordered semigroup S are key tools to describe the algebraic subsystems of S. Motivated by the study of prime fuzzy ideals in rings, semigroups and ordered semigroups, and also motivated by Kehayopulu and Tsingelis[10]'s work in ordered semigroups in terms of fuzzy subsets, in this paper we attempt to introduce and give a detailed investigation of quasi-prime fuzzy left ideals of an ordered semigroup S. We characterize quasi-prime fuzzy left ideals of S by ordered fuzzy points of S. Furthermore, we introduce the concept of fuzzy m-systems of an ordered semigroup S, and prove that a fuzzy left ideal f of S is quasi-prime if and only if $1 - f$ is a fuzzy m-system. Finally, we characterize the strongly semisimple ordered semigroups by quasi-prime fuzzy left ideals of S, and prove that S is strongly semisimple if and only if each fuzzy left ideal of S is the intersection of all quasi-prime left ideals of S containing it. As an application of the results of this paper, the corresponding results for semigroups (without ordered) are also obtained.

2 Preliminaries and Some Notations

Throughout this paper unless stated otherwise S stands for an ordered semigroup, that is, a semigroup S with an order relation " $\leq$ " such that $a \leq b$ implies $xa \leq xb$ and $ax \leq bx$ for any $x \in S$ (for example, see [11]). For convenience we use the notation $S^1 := S \cup \{1\}$, where $1 \cdot a = a \cdot 1 := a$ for all $a \in S$ and $1 \cdot 1 = 1$. A nonempty subset I of S is called a left (resp. right) ideal of S if

(1) $SI \subseteq I$ (resp. $IS \subseteq I$);

(2) If $a \in I, b \leq a$ with $b \in S$, then $b \in I$.

I is called an ideal of S if I is both a left and a right ideal of S (see [11]). Let L be a left ideal of S. L is called quasi-prime if for any two left ideals L_1, L_2 of S, $L_1L_2 \subseteq L$ implies $L_1 \subseteq L$ or $L_2 \subseteq L$; L is called quasi-semiprime if for any left ideal P of S such that $P^2 \subseteq L$, we have $P \subseteq L$ (see [11]).

For $H \subseteq S$, we define

$$(H] := \{t \in S \mid t \leq h \text{ for some } h \in H\}.$$

For $H = \{a\}$, we write $(a]$ instead of $(\{a\}]$. We denote by $L(a)$ the left ideal of S generated by $a \in S$. Then (see [12])

$$L(a) = (a \cup Sa] = (S^1a].$$

Lemma 2.1[12] *Let S be an ordered semigroup. Then the following statements hold*: