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Commutant Lifting for Optimal Control of Time-varying Systems*

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Abstract: This paper uses the commutant lifting theorem for representations of the nest algebra to deal with the optimal control of infinite dimensional linear time-varying systems. We solve the model matching problem and a certain optimal feedback control problem, each of which corresponds with one type of four-block problem. We also obtain a new formula for the optimal performance and prove the existence of an optimal controller.

Key words: optimal control, four block problem, commutant lifting, time-varying, nest algebra

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1 Introduction

The optimal control dealing with the problem of designing a controller to achieve a given prior optimal performance has been studied within various frameworks. Many results about time-invariant and time-varying optimal control for finite and infinite dimensional linear systems have been developed over the past few decades (see [1–3]). The computation of the optimal performance can be transferred into computing the block problem in the view of mathematics. For a discussion of the origins, Francis *et al.*[4] formulated the block problem related to H^∞ optimal control. The block problem associated to optimal control for time-varying systems was originally discussed by Feintuch *et al.* in [5–6]. A variety of methods have been used to study the optimal block problem. Among these methods, duality theory and the commutant lifting approach can obtain the existence of an optimal solution (see [7–11]). The commutant lifting theorem for operators has played an important role in dealing with H^∞ optimal block problems. Based on this theorem, Fioas *et al.*[3,10] indicated that the solutions to two types of the time-invariant four block problems are related to computing

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the norms of the certain operators. Ball[12] discussed the time-varying Nehari problem based on the commutant lifting theorem for the nest algebra. Recently, Djouadi[13] presented that the optimum of the mixed sensitivity problem, a special time-varying two block problem, is equal to the norm of the time-varying Hankel plus Toeplitz operator, and also proved the existence of an optimal controller by applying the nest algebra's commutant lifting theorem.

In this paper, we focus on applying the commutant lifting theorem to solve the time-varying model-matching problem and a certain optimal feedback control problem, which are two types of time-varying four block problem. In the case of linear time-invariant systems, the scalar and matrix-value model-matching problems have been reduced to Nehari-problems and have been studied in [1]. In [2], Feintuch showed that the optimal performance of the time-varying model-matching problem is equal to the supremum of a family operator norms. The optimal feedback control problem considered in [14] is a generalization of several optimal control problems, e.g., the weighted sensitivity and the disturbance attenuation problem. In fact, the optimum of this problem can be computed analogously to the computation of the measurement feedback control problem presented in [15]. But this method cannot obtain the attainment of a minimum. Here we prove the existence of an optimal controller and fit out that the optimum is equal to the norm of a certain operator.

2 Preliminaries

We recall some concepts and theorems (see [2, 12, 16]) used in this paper.

Let $\mathcal{H}$ and $\mathcal{K}$ be Hilbert spaces. "$\oplus$" stands for the direct sum of two spaces:

$$\mathcal{H} \oplus \mathcal{K} = \left\{ \begin{bmatrix} h \\ k \end{bmatrix} : h \in \mathcal{H},\ k \in \mathcal{K} \right\}.$$

If $\mathcal{K}$ is a closed subspace of $\mathcal{H}$, the orthogonal difference "$\ominus$" is defined as

$$\mathcal{H} \ominus \mathcal{K} = \{ h \in \mathcal{H} : \langle h,\ k \rangle = 0,\ k \in \mathcal{K} \}.$$

$\mathcal{B}(\mathcal{H}, \mathcal{K})$ is the Banach space of bounded linear operators from $\mathcal{H}$ to $\mathcal{K}$ with the operator norm

$$\|T\| = \sup_{x \in \mathcal{H},\ \|x\| \le 1} \|Tx\|.$$

$$\mathcal{B}(\mathcal{H}) = \mathcal{B}(\mathcal{H},\ \mathcal{H}).$$

"$*$" stands for the adjoint of an operator.

$$\ell^2 = \left\{ (x_0, x_1, \cdots, x_n, \cdots) : x_i \in \mathbb{C},\ \sum_{i=0}^{\infty} |x_i|^2 < \infty \right\}$$

is the Hilbert space with the inner product

$$\langle x,\ y \rangle = \sum_{i=0}^{\infty} x_i \bar{y}_i.$$

Let

$$P_{-1} = 0, \qquad P_{\infty} = I.$$

P_n denotes the usual truncation projection on ℓ^2, i.e.,

$$P_n(x_0, x_1, \cdots x_n, x_{n+1}, \cdots) = (x_0, x_1, \cdots, x_n, 0, 0, \cdots), \qquad n = 0, 1, \cdots$$