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Stability of Global Solution to Boltzmann-Enskog Equation with External Force*

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Abstract: In the presence of external forces depending only on the time and space variables, the Boltzmann-Enskog equation formally conserves only the mass of the system, and its entropy functional is also nonincreasing. Corresponding to this type of equation, we first give some hypotheses of its bicharacteristic equations and then get some results about the stablity of its global solution with the help of two new Lyapunov functionals: one is to describe interactions between particles with different velocities and the other is to measure the L^1 distance between two mild solutions. The former Lyapunov functional yields the time-asymptotic convergence of global classical solutions to the collision free motion while the latter is applied into the verification of the L^1 stability of global mild solutions to the Boltzmann-Enskog equation for a moderately or highly dense gas in the influence of external forces.
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1 Introduction

We are interested in the time-asymptotic behaviour and the Lyapunov stability of the global classical solution to the Enskog equation for a moderately or highly dense gas in the influence of external forces. As the generalization of the Boltzmann equation, the Enskog equation is a model first proposed by Enskog[1] in 1922 for a description of the dynamical behavior of the density of a moderately or highly dense gas. This is because the Boltzmann equation is no longer suitable for gases with high-density effects although it models dilute gases successfully.

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The Enskog equation is a partial differential integral equation of the hyperbolic type. There are some different versions of the Enskog equation in order that they formally satisfy some properties, such as entropy bound and consistence with irreversible thermodynamics (see [2–3]). We now take into account the so-called Boltzmann-Enskog equation, in the presence of external forces $E(t,x)$ depending on the time and space variables $t \in \mathbf{R}_+$ and $x \in \mathbf{R}^3$, as follows:

$$\frac{\partial f}{\partial t} + v \cdot \frac{\partial f}{\partial x} + E(t,x) \cdot \frac{\partial f}{\partial v} = Q(f) \tag{1.1}$$

for a one-particle distribution function $f = f(t,x,v)$ that depends on time $t \in \mathbf{R}_+$, the position $x \in \mathbf{R}^3$ and the velocity $v \in \mathbf{R}^3$, where Q is the collision operator whose form will be addressed below. Here and throughout this paper, $\mathbf{R}_+$ represents the positive side of the real axis including its origin and $\mathbf{R}^3$ denotes the three-dimensional Euclidean space.

The collision operator Q is expressed by the difference between the gain and loss terms respectively, and defined by

$$Q^+(f)(t,x,v) = a^2 \int_{\mathbf{R}^3 \times S^2_+} f(t,x,v')f(t,x-a\omega,w')B(v-w,\omega)\mathrm{d}\omega\mathrm{d}w, \tag{1.2}$$

$$Q^-(f)(t,x,v) = a^2 \int_{\mathbf{R}^3 \times S^2_+} f(t,x,v)f(t,x+a\omega,w)B(v-w,\omega)\mathrm{d}\omega\mathrm{d}w. \tag{1.3}$$

In (1.2)-(1.3), $S^2_+ = \{\omega \in S^2 : \omega \cdot (v-w) \geq 0\}$ is a subset of a unit sphere surface S^2 in $\mathbf{R}^3$, a is a diameter of hard sphere, ω is a unit vector along the line passing through the centers of the spheres at their interaction, (v', w') are velocities after collision of two particles having precollisional velocities (v,w), and

$$B(v-w,\omega) = \max\{0, (v-w)\cdot\omega\}$$

is the collision kernel.

The Boltzmann-Enskog equation (1.1) is a modification of Enskog's original work mentioned above and obeys only the conservation laws of mass in the presence of external forces. It is worth mentioning that the equation (1.1) still obeys the conservation laws of mass, momentum and energy under the assumption that $E(t,x) = 0$, that is, in the absence of external forces (see [4]).

As for the Boltzmann equation, two colliding particles obey the conservation laws of both kinetic momentum and energy as follows:

$$v + w = v' + w', \qquad v^2 + w^2 = {v'}^2 + {w'}^2. \tag{1.4}$$

This results in their velocity relations

$$v' = v - [(v-w)\cdot\omega]\omega, \qquad w' = w + [(v-w)\cdot\omega]\omega, \tag{1.5}$$

where $\omega \in S^2_+$. By (1.5), it follows that

$$B(v'-w', -\omega) = B(v-w,\omega).$$

The two postcollisional velocities given by (1.5) also have another expression as follows (see [5–6]):

$$v' = v - u_{\parallel}, \qquad w' = v - u_{\perp}. \tag{1.6}$$

where $u = v - w$, $u_{\parallel} = (u\cdot\omega)\omega$ and $u_{\perp} = u - u_{\parallel}$. Then the gain and loss terms (1.2) and