Communications in Mathematical Research
27(3)(2011), 207–214

K-controllability and Approximate K-controllability of Nonlinear Neutral Systems in Banach Spaces*

Lü Yue, Cai Hua and Liu Ming-ji
(*School of Mathematics, Jilin University, Changchun, 130012*)

Communicated by Li Yong

Abstract: In this paper, K-controllability and approximate K-controllability of nonlinear neutral differential equations in Banach spaces are studied. Sufficient conditions are established for each of these types of controllability. The results are obtained by using Leray-Schauder theory.

Key words: K-controllability, approximate K-controllability, m-accretive operator, Mild solution, preassigned response

2000 MR subject classification: 65M02, 65N15, 65R43

Document code: A

Article ID: 1674-5647(2011)03-0207-08

1 Introduction

In this paper we are concerned with the controllability of nonlinear neutral differential equations. In Section 3, we consider the K-controllability of the following system:

$$\frac{\mathrm{d}}{\mathrm{d}t}[x(t) + g(t, x(t))] + A(t, u)x(t) = B(t)u, \qquad t \in [0, T], \tag{1.1}$$

where $T > 0$ is fixed. The mapping $(t, u, x) \to A(t, u)x \in X$ is defined on the set $[0, T] \times D^u(A) \times D^x(A)$, where $D^u(A)$, $D^x(A)$ are constant subsets of the space X with $0 \in D^x(A)$, and $g : [0, T] \times X \to X$ is a continuous function. The symbol X denotes a real Banach space with normalized duality mapping J, $\|\cdot\|$ denotes the norm of X. The operators $B(t) : D(B) \subset U \to X$, where U is another real Banach space.

In Section 4, we consider the approximate K-controllability of the system

$$\frac{\mathrm{d}}{\mathrm{d}t}[x(t) + g(t, x(t))] + A(t)x(t) = B(t, x(t), u(t)), \qquad t \in [0, T], \ x(0) = 0, \tag{1.2}$$

where for every $t \in [0, T]$, $A(t) : D(A) \subset X \to X$, and $B : [0, T] \times X^2 \to X$ are given operators.

*Received date: Dec. 27, 2008.
Foundation item: The Young Scholar Foundation of Institute of Mathematics of Jilin University.

The concept of K-controllability or controllability with preassigned responses was introduced by Kartsatos and Mabry[1] in 1987. K-controllability problem was solved in various settings and some continuous controls were obtained in [2] and [3]. Recently Kartsatos[4] introduced the concept of approximate K-controllability. According to this theory, one attempts to control systems via responses which are chosen from a known set of smooth functions. As we just saw, the controllability problem is a problem of two unknowns: the control function $u(t)$, $t \in [0, T]$ and the response function $x(t)$, $t \in [0, T]$. In the theory of K-controllability, one attempts to solve the problem by first replacing the response $x(t)$, $t \in [0, T]$, by a preassigned response $f \in K(x_T)$. Here $K(x_T)$ is a known family of functions f which are strongly continuously differentiable on $[0, T]$ and such that

$$f(t) \in D(A), \qquad f(0) = 0, \qquad f(T) = x_T.$$

The K-controllability is very important to study the controllability of differential equation: firstly in a great variety of problems the system can be shown to be controllable, in the classical sense, although we have no information whatsoever about the solvability of the associated Cauchy problem; secondly the two-unknown problem has been reduced to a problem of one unknown, the control function $u(t)$, whose existence, at each $t \in [0, T]$, is a solution of the problem for a fixed function $f \in K(x_T)$.

The purpose of this paper is to extend the results of [3] and [4], by considering the nonlinear neutral systems which arises in various applications such as viscoelasticity, heat equations and many other physical phenomena (see [5], [6]).

2 Preliminaries

In this section, we describe necessary notations and definitions for the main result. Throughout this paper X denotes a real Banach space with normalized duality mapping J. The $\|\cdot\|$ denotes the norm of X as well as the norm of any other normed space under consideration. The symbol $B_u(0)$ is reserved for the ball of X, with center at 0 and radius $u > 0$.

An operator $T : D(A) \subset X \to X$ is accretive if for every $x, y \in D(A)$ there exists $x^* \in J(x - y)$ such that

$$\langle Tx - Ty,\ x^* \rangle \geq 0. \tag{2.1}$$

An accretive operator T is strongly accretive if 0 in the right-hand side of (2.1) is replaced by $\alpha\|x - y\|^2$, where $\alpha > 0$ is a fixed constant. An accretive operator T is called m-accretive if

$$R(T + \lambda I) = X$$

for every $\lambda > 0$, where I denotes the identity operator on X.

For an m-accretive operator T, the resolvent $J_\lambda : X \to D(t)$ are defined by

$$J_\lambda = (I + \lambda T)^{-1}$$

for all $\lambda \in (0, \infty)$. J_λ is a nonexpansive mapping on X for all $\lambda \geq 0$. Also, the operator

$$T_\lambda = (1/\lambda)(I - J_\lambda)$$

is a global Lipschitzian mapping with $T_\lambda x \in T J_\lambda x$, for every $x \in X$. For facts involving accretive operators, and other related concepts, the reader is referred to [7]–[10].