Communications
in
Mathematical
Research
26(4)(2010), 337–352

Practical Stability in the pth Mean of Stochastic Differential Equations with Discontinuous Coefficients*

Chen Xu-mei[1,2] and Zou Yong-kui[1]

(*1. School of Mathematics, Jilin University, Changchun, 130012,*)

(*2. Faculty of Science, Jiangsu University, Zhenjiang, Jiangsu, 210013*)

Abstract: In this paper, we give sufficient conditions to analyze the practical stability in the pth mean of stochastic differential equations with discontinuous coefficients. The Lyapunov-like function plays an important role in analysis. Some numerical computations are carried out to illustrate the theoretical results.

Key words: stochastic differential equation, practical stability in the pth mean, Lyapunov-like function

2000 MR subject classification: 60H10, 34D20, 65L07.

Document code: A

Article ID: 1674-5647(2010)04-0337-16

1 Introduction

It is well-known that the theory of stability in the sense of Lyapunov has been widely developed and has been vastly applied in many fields (see [1] and [2]). However, in some cases, a system may be stable or even asymptotically stable in the Lyapunov sense and it may still be completely useless in practice (see [3]), since the domain of attraction may not be large enough to allow the desired deviation to conceal out. On the other hand, a system may be unstable in the sense of Lyapunov and it may oscillate sufficiently near a state whose performance is acceptable in practice (see [4]). For example, many aircrafts and missiles behave in this manner. Thus, from practical consideration, a notion which is neither weaker nor stronger than Lyapunov stability is proposed by LaSalle and Lefschetz[3] and is developed by Michel[5], Bernfeld and Lakshmikantham[6], Martynyuk[7] and others. A systematic study on the practical stability is collected in the book [8]. Michel[9], Michel and Porter[10] and Zhai and Michel[11] also studied the practical stability of the deterministic discontinuous differential equations.

*Received date: Dec. 23, 2008.

Foundation item: The NSF (10671082) of China, the 985 Program of Jilin University, the Key Laboratory of Symbolic Computation and Knowledge Engineering of Ministry of Education, the Postgraduate Students Innovative Fund (20080239) of Jilin University, the Research Fund (10JDG020) for High-level Group of Jiangsu University.

In the development of modern mathematics, the theory of stochastic differential equation becomes even more important. It has been successfully applied to many fields involving nearly all aspects of reality. Many people have paid much attention to the qualitative properties of stochastic differential equations, such as the stability properties. Feng, Liu and Guo[12] applied the Lyapunov-like functions and the basic comparison principle to stochastic systems (see [13] and [14]) and established criteria for various types of practical stability in the pth mean of nonlinear stochastic systems. Ting[15] studied the almost sure practical stability with respect to some given continuous time-varying sets of the state space for stochastic differential systems by means of Lyapunov-like functions and comparison principle. Sathananthan and Suthaharan[16] generalized the concept of practical stability to the large-scale stochastic systems of the Itô-Doob type and established sufficient conditions for various types of practical stability in the pth mean.

In this paper, we concentrate on studying the practical stability and uniformly practical stability in the pth mean of stochastic differential equations with discontinuous coefficients, respectively.

This paper is organized as follows. In Section 2 we introduce the basic assumptions and then collect some definitions on practical stability in the pth mean. In Section 3 we study the practical stability and uniformly practical stability in the pth mean of the stochastic differential equations, respectively. In Section 4 we analyze some examples with numerical computation to illustrate the theoretical results.

2 Basic Assumptions

In this section we describe and discuss the basic assumptions for this work.

Let $\mathbf{R}^n$ denote the n-dimensional Euclidean space. $J = [t_0, t_0+T)$, where t_0 and $T \in \mathbf{R}^+$ and T may be finite or infinite. Let $(\Omega, \mathcal{F}, P)$ be a complete probability space and ξ be a random variable , $E(\xi)$ and $E(\|\xi\|^p)$ $(p \geq 1)$ denote the mean and pth mean of ξ, respectively.

A property is said to hold almost everywhere (abbrev. as a.e.) if the set of points where it fails is a set of measure zero.

In order to distinguish the fixed variable and random variable, we denote the fixed variable and random variable by x and X, respectively. Furthermore, we denote the random process by $X(t)$.

Consider a stochastic system described by the following n-dimensional stochastic differential equation:

$$\begin{cases} \mathrm{d}X(t) = b(X(t))\mathrm{d}t + \sigma(X(t))\mathrm{d}W(t), \\ X(t_0) = X_0, \end{cases} \tag{2.1}$$

where $\mathrm{d}X$ is a stochastic increment in the sense of Itô, $W(t)$ is an r-dimensional normalized Wiener process $(r \geq 1)$, the coefficients satisfy $b : \mathbf{R}^n \to \mathbf{R}^n$ (the drift vector), $\sigma : \mathbf{R}^n \to \mathbf{R}^{n\times r}$ and $a =: \sigma\sigma^T : \mathbf{R}^n \to \mathbf{R}^n \times \mathbf{R}^n$ (the diffusion matrix).

We assume that

(H1) The functions $b(\cdot)$ and $\sigma(\cdot)$ are locally bounded Borel measurable functions.