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On Existence of Local Solutions for a Hyperbolic System Modelling Chemotaxis with Memory Term

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Abstract. In this paper, we discuss the local existence of weak solutions for some hyperbolic parabolic systems modelling chemotaxis with memory term.The main methods we use are the fixed point theorem and semigroup theory.

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1 Introduction

The earliest model for chemosensitive movement has been developed by Keller and Segel in [1], which is described by the following parabolic system:

$$u_t = \nabla(\nabla u - \chi(v)\nabla v \cdot u), \tag{1.1}$$
$$\tau v_t = \triangle v + g(u,v), \tag{1.2}$$

where u represents the population density and v denotes the density of the external stimulus, χ is the sensitive coefficient,the time constant $0 \le \tau \le 1$ indicates that the spatial spread of the organisms u and the control signal v are on different time scales. The case $\tau \doteq 0$ corresponds to a quasi-steady-state assumption for the signal distribution. Since the KS model is designed to describe the behaviour of bacteria and bacteria aggregates, the question arises whether or not this model is able to show aggregation. Plenty of theoretical research uncovered exact conditions for aggregations and blow-up [2–16]. Besides, free boundary problems for the chemotaxis model are considered [18–25].

The possibility of blow-up of the solution for the KS model has been shown to depend strongly on space dimension. For instance, in the case of χ is constant and $g(u,v) =$

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$\gamma u - \delta u$, finite-time blow-up never occurs in 1D case [26] (unless there is no diffusion of the attractant v, [27,28]), but can always occur in N-dimensional cases for $N \geq 3$. For the 2D case, it depends on the initial data, i.e. there exists a threshold; if the initial distribution exceeds its threshold, then the solution blows up in finite time, otherwise the solution exists globally [29].

It is well-known that the movement behaviour of most species is usually guided by external signals, such as amoeba move upwards chemical gradients, insects approach towards light sources, the smell of a sexual partner makes it favourable to choose a certain direction. Motivated by these examples, if the external stimulus is based on the light (or the electromagnetic wave), then the control equation (1.2) should be replaced by a hyperbolic equation:

$$v_{tt} = \Delta v + g(u,v), \tag{1.3}$$

where, e.g. if the external signal is the electromagnetic field, then v would be voltage (in this case, ∇v denotes the electromagnetic field).

In this case, the full system for u and v become the following hyperbolic parabolic system:

$$u_t = \nabla(\nabla u - \chi(v)\nabla v \cdot u), \tag{1.4}$$

$$v_{tt} = \Delta v + g(u,v). \tag{1.5}$$

We introduce a memory term in equation (1.1), i.e.

$$u_t = \nabla(\nabla u - \chi(v) u \nabla v) + u^q \int_0^t u^p(\cdot,\tau)\mathrm{d}\tau, \tag{1.6}$$

where $p,q \geq 0$ are nonnegative constants.

Let $g(u,v) = -\gamma v + f(u)$, $\chi(v) = \chi$ is a constant, and supplement equations (1.5), (1.6) with some initial boundary conditions, then we obtain the following system

$$\begin{cases} u_t = \nabla(\nabla u - \chi u \nabla v) + u^q \int_0^t u^p \mathrm{d}\tau & \text{in } (0,T)\times\Omega, \\ v_{tt} = \Delta v + g(u,v) & \text{in } (0,T)\times\Omega, \\ \dfrac{\partial u}{\partial n} = \dfrac{\partial v}{\partial n} = 0 & \text{on } (0,T)\times\partial\Omega, \end{cases} \tag{1.7}$$

with initial data

$$u(0,\cdot) = u_0, \quad v(0,\cdot) = \varphi, \quad v_t(0,\cdot) = \psi, \quad \text{in } \Omega.$$

where $p \geq 0, q \geq 0$, $\Omega \subset R^N$, a bounded open domain with smooth boundary $\partial\Omega$, n is the unit outer normal on $\partial\Omega$ and χ is a nonnegative constant.

In this paper we would discuss the case of $p = 1, q = 0$.