

## A Second-Order Synchrosqueezing Transform with a Simple Form of Phase Transformation

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**Abstract.** To model a non-stationary signal as a superposition of amplitude and frequency-modulated Fourier-like oscillatory modes is important to extract information, such as the underlying dynamics, hidden in the signal. Recently, the synchrosqueezed wavelet transform (SST) and its variants have been developed to estimate instantaneous frequencies and separate the components of non-stationary multicomponent signals. The short-time Fourier transform-based SST (FSST for short) reassigns the frequency variable to sharpen the time-frequency representation and to separate the components of a multicomponent non-stationary signal. However FSST works well only with multicomponent signals having slowly changing frequencies. To deal with multicomponent signals having fast-changing frequencies, the second-order FSST (FSST2 for short) was proposed. The key point for FSST2 is to construct a phase transformation of a signal which is the instantaneous frequency when the signal is a linear chirp. In this paper we consider a phase transformation for FSST2 which has a simple expression than that used in the literature. In the study the theoretical analysis of FSST2 with this phase transformation, we observe that the proof for the error bounds for the instantaneous frequency estimation and component recovery is simpler than that with the conventional phase transformation. We also provide some experimental results which show that this FSST2 performs well in non-stationary multicomponent signal separation.

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## 1. Introduction

To model a non-stationary signal  $x(t)$  as

$$x(t) = A_0(t) + \sum_{k=1}^K x_k(t), \quad x_k(t) = A_k(t)e^{i2\pi\phi_k(t)} \quad (1.1)$$

with  $A_k(t), \phi'_k(t) > 0$  is important to extract information hidden in  $x(t)$  since most real signals such as EEG and bearing signals can be formulated as (1.1), and its trend  $A_0(t)$ , instantaneous amplitudes  $A_k(t)$  ( $k \geq 1$ ) and instantaneous frequencies  $\phi'_k(t)$  can be used to describe the underlying dynamics of  $x(t)$ . Thus the representation (1.1) of non-stationary signals has been used in many applications including geophysics (seismic wave), atmospheric and climate studies, oceanographic studies, medical data analysis and speech recognition, see for example [10]. The empirical mode decomposition (EMD) [9] is a widely used method to separate a signal as a sum of finitely many intrinsic mode functions (IMFs) and represent the signal in the form of (1.1) by the Hilbert analysis. EMD is a data-driven decomposition algorithm and it was studied by many researchers and has been used in many applications, see, e.g., [7, 17, 20, 27, 31, 34, 36, 37, 41]. However EMD hardly distinguishes two close IMFs and sometimes it leads to false components.

Recently the continuous wavelet transform-based synchrosqueezing transform (WSST) was developed in [6] as an alternative EMD-like tool to separate the components of a non-stationary multicomponent signal. In addition, the short-time Fourier transform-based SST (FSST) was also proposed in [30] and further studied in [23, 35] for this purpose. SST has been proved to be robust to noise and small perturbations [11, 21, 29]. However SST does not work well for multicomponent signals having fast changing frequencies.

To provide sharp representations for signals with significantly frequency changes, the second-order FSST (FSST2) and the second-order WSST (WSST2) were introduced in [22, 24], and the theoretical analysis of them was carried out in [1] and [26], respectively. The second-order SST improves the concentration of the time-frequency representation. The higher-order FSST is presented in [18, 25], which aims to handle signals containing more general types. Other SST related methods include the generalized WSST [13], a hybrid empirical mode decomposition-SST computational scheme [5], the synchrosqueezed wave packet transform [38], the demodulation-transform based SST [12, 32, 33], signal separation operator [4], vertical synchrosqueezing [8] and empirical signal separation algorithm [16]. In addition, the synchrosqueezed curvelet transform for two-dimensional mode decomposition was introduced in [40] and the statistical analysis of synchrosqueezing transforms has been studied in [39]. Furthermore, the SST with a window function having a changing parameter was proposed in [28] and the FSST with the window function containing time and frequency parameters was studied in [2]. Very recently the authors of [14, 15] considered the second-order adaptive FSST and WSST with a time-varying parameter. They obtained the