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Determination of Random Periodic Structures in Transverse Magnetic Polarization

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Abstract. Consider an inverse problem that aims to identify key statistical properties of the profile for the unknown random perfectly conducting grating structure by boundary measurements of the diffracted fields in transverse magnetic polarization. The method proposed in this paper is based on a novel combination of the Monte Carlo technique, a continuation method and the Karhunen-Loève expansion for the uncertainty quantification of the random structure. Numerical results are presented to demonstrate the effectiveness of the proposed method.

AMS subject classifications: 78A46, 65C30, 65N21

Key words: Random periodic structure, inverse scattering, Karhunen-Loève expansion, Monte Carlo-continuation-uncertainty quantification method.

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1 Introduction

The direct and inverse scattering problems in periodic structures have received considerable attentions due to diverse sets of applications, especially in the design and manufacture of optical elements including correction lenses, sensors, solar cells and surface plasmons [4,13,18,19,23,32]. In this paper, we consider the

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inverse scattering of a time-harmonic electromagnetic plane wave by a periodic structure which is usually regarded as a grating in diffractive optics.

There are two fundamental polarizations for electromagnetic fields: transverse electric (TE) polarization and transverse magnetic (TM) polarization. The direct and inverse scattering problems in periodic structures for TE and TM polarization have been studied both numerically and mathematically. For the mathematical studies of the existence and uniqueness of the diffraction of a time harmonic wave modeled by a generalized Helmholtz equation in TE and TM polarization, we refer to [2,7,8,27] and references therein. A comprehensive review of diffractive optics in mathematics and computation can be found in [6,10,30]. Numerically, most of the studies are concerned with TE polarization, for which many numerical methods have been developed in [1,15,24,25,33] to solve the inverse scattering problems. A method of coupling of finite element and boundary integral equations for the solutions of direct and inverse electromagnetic scattering in both transverse electric and magnetic polarization cases was proposed in [9,28]. An efficient continuation method to capture the grating profile with multiple frequency data was developed in [12] and further extended to the case of phaseless data in [11]. Recently, Qu *et al.* [31] proposed a novel integral equation for direct and inverse scattering by a locally perturbed infinite plane on which a Neumann boundary condition is imposed.

This paper is devoted to the numerical solution of the inverse scattering by a periodic structure for TM polarization, i.e., given the incident field, determine the periodic structure from the measured diffracted field at a constant distance from the structure.

By far, most of the research in the area of diffractive optics has assumed that the periodic grating profile is deterministic and only the noise level of measurements is considered for the inverse problem. In reality, however, one has to deal with the uncertainty of grating structures, including its manufacturing defects, some possible damage due to long-term usage and so on. Actually, surface roughness measurement is of great significance for the functional performance evaluation of machined parts and the design of micro-optical components. Thus a natural question arises: in addition to the noise level of the measured data, how does one reconstruct the unknown grating profile by considering the uncertainty of gratings? The real challenge is how to characterize the grating profile with uncertainties appropriately based on the measured data. What makes the reconstruction extremely difficult is the nonlinearity between the measured diffracted field and the unknown grating structure. For the direct random scattering problem, there have been many studies in various numerical methods [3,35]. Feng *et al.* [21] proposed an efficient Monte Carlo-transformed field expansion method for the