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A PML Method for Electromagnetic Scattering from Two-dimensional Overfilled Cavities*

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Abstract: In this paper, we consider electromagnetic scattering problems for two-dimensional overfilled cavities. A half ringy absorbing perfectly matched layer (PML) is introduced to enclose the cavity, and the PML formulations for both TM and TE polarizations are presented. Existence, uniqueness and convergence of the PML solutions are considered. Numerical experiments demonstrate that the PML method is efficient and accurate for solving cavity scattering problems.

Key words: overfilled cavity, scattering, TM polarization, TE polarization, perfectly matched layer (PML), DtN operator

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1 Introduction

The problem of calculating electromagnetic scattering from open cavities, such as jet inlet and exit of airplane, is of high interest in certain military and defense applications, because open cavities usually cause strong radar echo and can be easily found. Researchers in the engineering community have raised many methods to solve scattering problems involving cavities filled with penetrable material, which include high and low frequency method (see [1]–[3]), the method of moment (see [4] and [5]) and Finite element-Boundary integral equation method (see [6]–[8]). Analysis of cavity scattering problems can also be found in mathematic area (see [9]–[12]). It is a common assumption that the cavity opening coincides with the aperture on an infinite ground plane, and hence simplifying the modelling of the exterior (to the cavity) domain. This limits the applications of these methods since many cavity openings are not planar. For determining the fields scattered by overfilled cavities, we have to find new ways.

In [13], a mathematical model characterizing the scattering by overfilled cavities was developed and proved to be well posed. For solving overfilled cavity scattering problems,

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[14]–[16] presented a method and [16] gave the corresponding mathematical analysis. Although this method reduces the infinite computational domain exactly to a finite one, the numerical implementation is difficult, because the boundary condition imposed on the artificial semicircle includes the DtN operator.

Zhang and Ma presented a PML method in [17] to solve cavity scattering problems which can also be used to solve overfilled cavity scattering problems. But they only consider the case of TM polarization. Basing on the works of Aihua Wood on overfilled cavity scattering (see [14]–[16]) and Chen *et al.*[18] on PML technique for obstacle scattering, by introducing symmetrical prolongation, we present a PML method to solve cavity scattering problems, for both TM and TE polarizations. Our method also reduces the infinite computational domain to a finite one that enclosed by a half ringy PML layer. The PML formulations for both TM polarization and TE polarization of cavity scattering are presented. We establish the existence and uniqueness of the PML solutions, and we also prove that the PML solutions converge exponentially to the exact solutions derived by the DtN method. Numerical computations demonstrate that our method is effective and its numerical computing is easy to be implemented on the PDE toolbox of MATLAB or other finite element softwares.

2 Problem Setting

Let $\Omega \subset \mathbb{R}^2$ be the cross-section of a z-invariant trough in the infinite ground plane such that its fillings protrude above the ground plane. Denote S as the cavity wall, and Γ the cavity aperture so that $\partial\Omega = S \cup \Gamma$. The infinite ground plane excluding the cavity opening is denoted as Γ_{ext}, the infinite homogenous region above the cavity as $\mathcal{U} = \mathbb{R}^2_+ \backslash \Omega$.

Given the incident electromagnetic wave $(\mathbf{E}^{\text{i}},\ \mathbf{H}^{\text{i}})$, we wish to determine the resulting scattering field $(\mathbf{E}^{\text{s}},\ \mathbf{H}^{\text{s}})$.

Due to the uniformity in the z-axis, the scattering problem can be decomposed into two fundamental polarizations: transverse magnetic (TM) and transverse electric (TE). Its solution can be expressed as a linear combination of the solutions to TM and TE problems.

In the TM polarization, the magnetic field $\mathbf{H}$ is transverse to the z-axis so that $\mathbf{E}$ and $\mathbf{H}$ are of the form

$$\mathbf{E} = (0, 0, E_z), \qquad \mathbf{H} = (H_x, H_y, 0).$$

In this case, by setting $u = E_z$, we can determine $\mathbf{E}$ and $\mathbf{H}$ by u which satisfies the scattering problem with the following form:

$$(\text{TM}) \begin{cases} \Delta u + k^2 \varepsilon_r u = 0 & \text{in } \Omega \cup \mathcal{U}, \\ u = 0 & \text{on } S \cup \Gamma_{\text{ext}}. \end{cases}$$

u naturally satisfies the continuity conditions on Γ:

$$u|_{\Gamma+} = u|_{\Gamma-}, \qquad \frac{\partial u}{\partial n}\Big|_{\Gamma+} = \frac{\partial u}{\partial n}\Big|_{\Gamma-}.$$

$\varepsilon_r = \varepsilon/\varepsilon_0$ is the relative electric permittivity and k is the free space wave number. We assume $\text{Re}\varepsilon_r \geq \alpha > 0$, $\text{Im}\varepsilon_r \geq 0$, and $\varepsilon_r \in L^\infty(\Omega)$. The homogeneous region $\mathcal{U}$ above the