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Pre-image Variational Principle for Subadditive Sequence Functions*

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Abstract: In this paper we define the pre-image topological pressure for a sequence of subadditive continuous functions on the compact metric space. And we also give a subadditive pre-image variational principle under a very weak condition.

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1 Introduction

The topological pressure and variational principle for additive condition was first presented by Ruelle[1]. These notions together with equilibrium states play an important role in statistic mechanics, ergodic theory and dynamical systems (see [2]–[4]). For non-additive case, Falconer[5] conducted the subadditive variational principle on mixing repellers, Barreira[6] introduced the non-additive variational principle on an arbitrary subset of compact metric space under a mild condition. In [7], Barreira established a variational principle on the repeller for almost additive sequences and discussed the existence and uniqueness of equilibrium and Gibbs measures. Feng developed a variational principle for a sequence of functions on a subshift of finite type in the context of multifractal formalism associated to iterated function systems with overlaps (see [8] and [9]). Murmmert[10] gave the variational principle under some assumption for the toplogical pressure defined on a subset of compact metric space for an almost additive sequence. Cao[11] defined the topological pressure for

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subadditive sequence and set up a variational principle without any additional assumptions.

In recent years, the pre-image structure of a map has been studied by many authors (see [12]–[17]). Fiebig *et al.*[16] studied the relation between the classical topological entropy and the dispersion of pre-images. Cheng and Newhouse[17] defined the pre-image entropies which are similar to the standard notions of topological and measurable entropies. They obtain a variational principle for entropy which is much more like the one in classical case. Zeng[18] presented the notion of pre-image pressure and investigated the relationship with invariant measures. A variational principle for pre-image pressure is also established there, which generalized Cheng's result to pre-image pressure.

In this paper, we present the sub-additive version of pre-image pressure and the corresponding variational principle. We generalize Zeng's results to sub-additive situation. In fact, we set up a variational principle between the sub-additive pre-image pressure, the pre-image entropy and some functions about invariant measure which like the Lyapunove exponents. Our restricted condition is very weak. We only assume that the pre-image pressure is not $-\infty$. As to the case of $-\infty$, the defined pre-image pressure is tightly relative to the condition $\Phi^*(\mu) = -\infty$ for all invariant measure μ. The method we used is still in the frame of Misiurewicz's elegant proof (see [19]), which has been used by many authors (see [20]). Both Ledrappier's method for proving relative variational principle (see [20]) and Cao's technique for arguing subadditive variational principle (see [11]) are used in the argument of our theorem. It should be pointed out that our approach is different from Zeng's for additive case (see [18]) and Cheng's for pre-image entropy (see [17]) in the part 1 of the proof.

This paper is organized as follows. In Section 2, we give the preliminary, which involves the definitions of pre-image entropy and subadditive pre-image pressure and a lemma. In Section 3, we state and prove the subadditive pre-image variational principle after proposing a needed lemma.

2 Preliminaries

Let (X, d) denote a compact metric space with metric d. Let $\mathcal{B}$ be the Borel σ-algebra of X and let $\mathcal{M}(X)$ be the collection of all probability measures defined on the measurable space $(X, \mathcal{B})$. Let $f : X \to X$ be a continuous map on X and let $\mathcal{M}(X, f)$ be the sets of f-invariant Borel probability measures. Obviously, $\mathcal{M}(X, f) \subset \mathcal{M}(X)$.

Set

$$\mathcal{B}^- = \bigcap_{n=0}^{\infty} f^{-n}\mathcal{B}.$$

For any finite partition α, $H_\mu(\alpha^n|\mathcal{B}^-)$ (see [21]) is a non-negative subadditive sequence for $\mu \in \mathcal{M}(X, f)$, where

$$\alpha^n = \bigvee_{i=0}^{n-1} f^{-i}\alpha.$$