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Further Extension of a Theorem of Values Distribution of Meromorphic Function $f^{(k)}f^{n*}$

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Abstract: In this paper we derive the fundamental inequality of the theorem of meromorphic functions. It extends some results of Yi Hong-xun *et al.* As one of its application, we then study the value distribution of $f^{(k)}f^n - c(z)$.
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1 Introduction and Main Results

Let $\mathbb{C}$ denote the whole complex plane. Throughout this paper suppose that f is a transcendental meromorphic function in $\mathbb{C}$ and $a \in \mathbb{C}$ is a finite complex number. The function $c(z)$ is said to a small function of f provided that

$$T(r, c(z)) = o(T(r, f)) \qquad (r \to \infty)$$

or

$$c(z) \equiv \infty.$$

In the following we use standard notations of Nevanlinna's Theory and its some fundamental results (see [1]). In particular,

$$S(r, f) = o(T(r, f)) \qquad (r \to \infty)$$

except for a finite linear measure of the set of the value r. In addition, we suppose that k is a positive integer, denote by $N_{k)}\left(r, \dfrac{1}{f}\right)$ (counting multiplicity) the counting function of the multiplicity of all zeros of f, all of whose zeros have the multiplicity at most k. $N_{(k+1}\left(r, \dfrac{1}{f}\right)$ (counting multiplicity) is denoted to the counting function of the multiplicity of all zeros

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of f, all of whose zeros have the multiplicity at least $k+1$. Finally, $\overline{N}_{k)}\left(r,\ \frac{1}{f}\right)$ and $\overline{N}_{(k+1}\left(r,\ \frac{1}{f}\right)$ are defined to the counting functions (ignoring multiplicity).

In 1989, Yi[2] proved the following result.

Theorem A *Let f be a transcendental meromorphic function. Then*

$$2T(r,f) \le \overline{N}\left(r,\ \frac{1}{f'f-a}\right) + 2\overline{N}\left(r,\ \frac{1}{f}\right) + \overline{N}(r,f) + S(r,f).$$

In 1993, Ye[3] proved the following theorem.

Theorem B *Let f be a transcendental meromorphic function and n be a positive integer. Then*

$$(n+1)T(r,f) \le \overline{N}\left(r,\ \frac{1}{f'f^n-a}\right) + 2\overline{N}\left(r,\ \frac{1}{f}\right) + \overline{N}(r,f) + S(r,f).$$

In 2007, Dou and Wu[4] improved Theorem B as follows.

Theorem C *Let f be a transcendental meromorphic function and n be a positive integer. If $c(z)$ is a small function of f, then*

$$(n+1)T(r,f) \le \overline{N}\left(r,\ \frac{1}{f'f^n-c(z)}\right) + 2\overline{N}\left(r,\ \frac{1}{f}\right) + \overline{N}(r,f) + S(r,f).$$

In this paper we are inspired by the above results to get the following more general result.

Theorem 1.1 *Let f be a transcendental meromorphic function with all of whose zeros have multiplicity at least k and n, k be two positive integers. If $c(z)$ is a small function of f, then*

$$(n+1)T(r,f) \le \overline{N}\left(r,\ \frac{1}{f^{(k)}f^n-c(z)}\right) + (k+1)\overline{N}\left(r,\ \frac{1}{f}\right) + \overline{N}(r,f) + S(r,f).$$

We can deduce from Theorem 1.1 the following result.

Theorem 1.2 *Let f be a transcendental meromorphic function with all of whose zeros have multiplicity at least $k+1$ and n, k be two positive integers. If f has only finitely many simple poles and $c(z)$ is a small function of f, then $f^{(k)}f^n-c(z)$ has infinitely many zeros.*

2 Lemmas Needed in Proofs of Theorems

Suppose that f is non-constant and meromorphic and k is a positive integer. We denote by

$$M[f] = f^{n_0}(f')^{n_1}\cdots(f^{(k)})^{n_k}$$

a differential one-polynomial of f, where n_0, n_1, $\cdots$, n_k are zeroes or positive integers,

$$\gamma_M := \sum_{j=0}^{k} n_j$$