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Dependence of the Blow-up Time with Respect to Parameters for Semilinear Parabolic Equations with Nonlinear Memory*

Huang Hui[1,2], Guan Lü-tai[1] and Zhu Qing-yong[1]
(*1. Department of Mathematics, Sun Yat-sen University, Guangzhou, 510275*)
(*2. Department of Computation, Guangdong Commercial College, Guangzhou, 510320*)

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Abstract: In this paper we discuss the bounds for the modulus of continuity of the blow-up time with respect to three parameters of λ, h, and p respectively for the initial boundary value problem of the semilinear parabolic equation.

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1 Introduction

Let $\Omega \subset R^n$ be open and smoothly bounded. Suppose that $u(x,t)$ is well defined on $\overline{\Omega}\times[0,T)$ for $T > 0$. If

$$\lim_{t\to T}\sup ||u(x,t)||_\infty = \infty,$$

we call T the blow-up time of $u(x,t)$ and $u(x,t)$ blows up at time T. In recent years, there have been a large number of papers concerned with the blow-up properties of solutions of parabolic equations (see [1]–[5]).

Consider the following initial boundary value problem

$$\begin{cases} u_t = \lambda\Delta u + u^p, & (x,t) \in \Omega \times (0,T), \\ u = 0, & (x,t) \in \partial\Omega \times (0,T), \\ u(x,0) = u_0(x) = \Phi(x) + hf(x), & x \in \Omega. \end{cases} \tag{1.1}$$

It is well known that the blow-up time of solutions of a parabolic problem is continuous with respect to the initial data u_0 (see [6] and [7]).

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In [8], Groisman and Rossi established the dependence of the blow-up time T of problem (1.1) on the parameters p, h and λ, respectively. The bound of the form

$$|T - T_h| \le C|h|^\gamma \tag{1.2}$$

was also obtained in [8].

In [2], Li and Xie investigated the following semilinear parabolic equation

$$u_t = \Delta u + u^q \int_0^t u^p(x,s)\mathrm{d}s, \qquad x \in \Omega, \quad t > 0 \tag{1.3}$$

with homogeneous Dirichlet boundary value conditions. It was proved that u satisfies

$$c(T-t)^{-\frac{2}{p-1}} \le \max_{x\in\overline{\Omega}} u(x,t) \le C(T-t)^{-\frac{2}{p-1}} \qquad \text{as } t \to T \tag{1.4}$$

if for each solution u of equation (1.3) there exists $t_0 \in [0, T)$ such that $u_t(x, t_0) \ge 0$ for all $x \in \Omega$. In [4], the authors investigated the monotonicity and blow-up rate of solutions of equation (1.3) with $q = 0$.

Motivated by [2], [4] and [8], we consider the following initial boundary value problem of semilinear parabolic equation

$$\begin{cases} u_t = \lambda\Delta u + \displaystyle\int_0^t u^p(x,s)\mathrm{d}s, & (x,t) \in \Omega \times (0,T), \\ u = 0, & (x,t) \in \partial\Omega \times (0,T), \\ u(x,0) = u_0(x) = \Phi(x) + hf(x), & x \in \Omega, \end{cases} \tag{1.5}$$

where $p > 1$, $\lambda > 0$ and $h \in R$ are parameters, $u_0 \ge 0$ is a sufficiently regular function with $u_0|_{\partial\Omega} = 0$ and there exists $t_0 \in [0, T)$ such that $u_t(x, t_0) \ge 0$ for all $x \in \Omega$.

We are interested in the study of the dependence of the blow-up time T of solutions of problem (1.5) on the parameters λ, h and p, respectively. The purpose of this paper is to prove that there exists a modulus of continuity with respect to the initial condition h, to the diffusion coefficient λ and to the reaction power p, respectively. The proof of the main results of this paper is based upon the estimates of the first time where two different solutions spread. These estimates are based on the rate at which solutions blow up.

The remainder of this paper is organized as follows. In Section 2, the estimate of the modulus of continuity of the blow-up time for the problem (1.5) with respect to the parameter p is made. The estimate of the blow-up time with respect to the parameter h is made in Section 3. Finally in Section 4, we establish a similar estimate with respect to the parameter λ.

2 Perturbations in the Nonlinear Source

We first pay attention to the dependence of the blow-up time T with respect to the parameter p. We consider the following problems

$$\begin{cases} u_t = \Delta u + \displaystyle\int_0^t u^p(x,s)\mathrm{d}s, & (x,t) \in \Omega \times (0,T), \\ u(x,t) = 0, & (x,t) \in \partial\Omega \times (0,T), \\ u(x,0) = u_0(x) = \Phi(x), & x \in \Omega, \end{cases} \tag{2.1}$$