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Weighted Endpoint Estimates for Multilinear θ-type Calderón-Zygmund Operator*

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Abstract: In this paper, weighted endpoint boundedness of multilinear θ-type Calderón-Zygmund operator is obtained.

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1 Introduction

Let T be a Calderón-Zygmund operator. Coifman *et al.*[1] stated that the commutator

$$[b,\ T] = b(Tf) - T(bf), \qquad b \in BMO(\mathbf{R}^n)$$

is bounded on $L^p(\mathbf{R}^n)$ for $1 < p < \infty$. Chanillo[2] proved a similar result when T is replaced by the fractional integral operator. In [3], the boundedness of the commutator for the extreme values of p is obtained. Recently, Liu[4] proved the weighted boundedness of multilinear operators related to some non-convolution operators for the extreme cases. In [5], Yabuta introduced θ-type Calderón-Zygmund operator to facilitate study of pseudo-differential operators. In this paper, we obtain the weighted endpoint boundedness for multilinear θ-type Calderón-Zygmund operator.

Throughout this paper, Q denotes a cube of $\mathbf{R}^n$ with sides parallel to the axes. For a cube Q and a locally integrable function f, let

$$f(Q) = \int_Q f(x)\mathrm{d}x, \qquad f_Q = |Q|^{-1}\int_Q f(x)\mathrm{d}x$$

and

$$f^\sharp(x) = \sup_{x\in Q} |Q|^{-1}\int_Q |f(y) - f_Q|\mathrm{d}y.$$

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Moreover, for a weight function ω, f is said to belong to $BMO(\omega)$ if $f^\sharp \in L^\infty(\omega)$ and define

$$\|f\|_{BMO(\omega)} = \|f^\sharp\|_{L^\infty(\omega)}.$$

If $\omega = 1$, then we denote that

$$BMO(\omega) = BMO(\mathbf{R}^n).$$

Also, we give the concepts of the atom and weighted H^1 space. A function a is called an H^1 atom if there exists a cube Q such that a is supported on Q,

$$\|a\|_{L^\infty(\omega)} \le \omega(Q)^{-1}$$

and

$$\int a(x)\mathrm{d}x = 0.$$

It is well known that weighted Hardy space $H^1(\omega)$ has the atomic decomposition characterization (see [6]).

Let m be a positive integer and A be a function on $\mathbf{R}^n$. We denote

$$R_{m+1}(A;\ x,y) = A(x) - \sum_{|\alpha|\le m} \frac{1}{\alpha!} D^\alpha A(y)(x-y)^\alpha$$

and

$$Q_{m+1}(A;\ x,y) = R_{m+1}(A;\ x,y) - \sum_{|\alpha|=m} \frac{1}{\alpha!} D^\alpha A(x)(x-y)^\alpha.$$

Definition 1.1 *Let θ be a nonnegative nondecreasing function on $\mathbf{R}^+ = (0,\infty)$ satisfying*

$$\int_0^1 \frac{\theta(t)}{t}|\lg t|\mathrm{d}t < \infty.$$

A kernel $K(x,y) \in L^1_{loc}(\mathbf{R}^n \times \mathbf{R}^n \setminus \{(x,y):\ x=y\})$ is called a θ-type Calderón-Zygmund kernel if

$$|K(x,y)| \le C|x-y|^{-n} \tag{1.1}$$

for $x \ne y$, and

$$|K(x,y) - K(x,z)| + |K(y,x) - K(z,x)| \le C\theta\left(\frac{|y-z|}{|x-z|}\right)|x-z|^{-n} \tag{1.2}$$

for $2|y-z| \le |x-z|$.

The θ-type Calderón-Zygmund operator associated with the above kernel $K(x,y)$ is defined by

$$Tf(x) = p.v. \int_{\mathbf{R}^n} K(x,y)f(y)\mathrm{d}y. \tag{1.3}$$

Definition 1.2 *The multilinear θ-type Calderón-Zygmund operator is defined by*

$$T^A f(x) = \int \frac{R_{m+1}(A;\ x,y)K(x,y)}{|x-y|^m} f(y)\mathrm{d}y,$$

where $K(x,y)$ is a θ-type Calderón-Zygmund kernel.

We also consider the variant of T^A, which is defined by

$$\widetilde{T}^A f(x) = \int \frac{Q_{m+1}(A;\ x,y)K(x,y)}{|x-y|^m} f(y)\mathrm{d}y.$$