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Normal Functions Concerning Shared Values*

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Abstract: In this paper we discuss normal functions concerning shared values. We obtain the follow result. Let $\mathcal{F}$ be a family of meromorphic functions in the unit disc Δ, and a be a nonzero finite complex number. If for any $f \in \mathcal{F}$, the zeros of f are of multiplicity, f and f' share a, then there exists a positive number M such that for any $f \in \mathcal{F}$, $(1 - |z|^2)\dfrac{|f'(z)|}{1 + |f(z)|^2} \leq M$.

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1 Introduction

Let f and g be two meromeorphic functions in a domain D of the complex plane, and a, b be two complex numbers. We say that $f = a \Longleftrightarrow g = b$, if $\bar{E}_f(a) = \bar{E}_g(b)$. Especially, we say f and g share the value a, if $\bar{E}_f(a) = \bar{E}_g(a)$. We say that $f = a \Longrightarrow g = b$, if $\bar{E}_f(a) \subseteq \bar{E}_g(b)$. Here

$$\bar{E}_f(a) = f^{-1}(a) \cap D = \{z \in D : f(z) = a\}.$$

A meromorphic function f on $\mathbb{C}$ is called a normal function if there exists a positive number M, such that

$$f^{\#}(z) = \frac{|f'(z)|}{1 + |f(z)|^2} \leq M.$$

A meromorphic function f in the unit disc Δ is called a normal function if there exists a positive number M such that

$$(1 - |z|^2)f^{\#}(z) = (1 - |z|^2)\frac{|f'(z)|}{1 + |f(z)|^2} \leq M.$$

Schwick[1] proved the following theorem.

Theorem A *Let $\mathcal{F}$ be a family of meromorphic functions in the unit disc Δ, and a, b, c be distinct complex numbers. If for any $f \in \mathcal{F}$, f and f' share a, b and c, then $\mathcal{F}$ is a*

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normal family in the unit disc Δ.

Pang and Zalcman[2] improved the Theorem A and obtained the following result.

Theorem B *Let* $\mathcal{F}$ *be a family of meromorphic functions in the unit disc* Δ*, and* a, b *be two distinct complex numbers. If for any* $f \in \mathcal{F}$, f *and* f' *share* a *and* b*, then* $\mathcal{F}$ *is normal in the unit disc* Δ.

In 2000, Pang[3] found a connection between normal family and normal function. He proved the following theorem.

Theorem C *Let* $\mathcal{F}$ *be a family of meromorphic functions in the unit disc* Δ*, and* a, b, c *be distinct finite complex numbers. If for any* $f \in \mathcal{F}$, f *and* f' *share* a, b *and* c*, then there exists a positive number* $M = M(a, b, c)$ *such that for any* $f \in \mathcal{F}$,

$$(1 - |z|^2)\frac{|f'(z)|}{1 + |f(z)|^2} \leq M.$$

It is natural to ask whether for Theorem B one can obtain a conclusion like Theorem C. In this paper we study the problem and obtain the following theorems.

Theorem 1.1 *Let* f *be a meromorphic function on* $\mathbb{C}$*, and* a *be a nonzero finite complex number. If the zeros of* f *are of multiple,* f *and* f' *share* a*, then* f *is a normal function on* $\mathbb{C}$.

Theorem 1.2 *Let* $\mathcal{F}$ *be a family of meromorphic functions in the unit disc* Δ*, and* a *be a nonzero finite complex number. If for any* $f \in \mathcal{F}$*, the zeros of* f *are of multiplicity,* f *and* f' *share* a*, then there exists a positive number* M *such that for any* $f \in \mathcal{F}$,

$$(1 - |z|^2)\frac{|f'(z)|}{1 + |f(z)|^2} \leq M.$$

2 Some Lemmas

Lemma 2.1[4] *Let* $\mathcal{F}$ *be a family of meromorphic functions in the unit disc* Δ*, all of whose zeros have multiplicity at least* k*. Suppose that there exists* $A \geq 1$ *such that* $|f^{(k)}(z)| \leq A$ *whenever* $f(z) = 0$*. Then if* $\mathcal{F}$ *is not normal, there exist, for each* $0 \leq \alpha \leq k$,

(a) *a number* $0 < r < 1$;

(b) *points* z_n, $|z_n| < r$;

(c) *functions* $f_n \in \mathcal{F}$; *and*

(d) *positive numbers* $\rho_n \to 0$

such that

$$\rho_n^{-\alpha} f_n(z_n + \rho_n \xi) = g_n(\xi) \to g(\xi)$$

locally uniformly with respect to the spherical metric, where g *is a nonconstant meromorphic function on* $\mathbb{C}$*, all of whose zeros have multiplicity at least* k*, such that*

$$g^{\#}(\xi) \leq g^{\#}(0) = kA + 1.$$