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Theory and Applications of Distinctive Conformable Triple Laplace and Sumudu Transforms Decomposition Methods

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Abstract. This article presents some important results of conformable fractional partial derivatives. The conformable triple Laplace and Sumudu transform are coupled with the Adomian decomposition method where a new method is proposed to solve nonlinear partial differential equations in 3-space. Moreover, mathematical experiments are provided to verify the performance of the proposed method. A fundamental question that is treated in this work: is whether using the Laplace and Sumudu transforms yield the same results? This question is amply answered in the realm of the proposed applications.

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1 Introduction

In recent years, it has been found that fractional partial differential equations (FPDEs) are very important for modelling many Real-life sciences and engineering applications, such as fluid dynamics, biology of mathematics, electrical circuits, optics, Quantum Mechanics, etc. [1-3]. Many definitions of fractional derivatives and integrals, such as Rizez, Weyl, Riemann-Liouville, Caputo, Hadamard and so on, have been mentioned in literature. These forms of fractional derivatives have many peculiar properties such

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as not all two functions follow chain rule, product, and quotient rule, these significant properties lead to some flaws in physics and engineering applications. In 2014, Khalil et. al [4] introduced a new type of derivative called the conformable fractional derivative (CFD) which satisfies the classical properties of the known derivative. The conformable fractional derivative of function $f:(0,\infty)\to R$, of order $\alpha\in(0,1]$ is defined by,

$$T^{\alpha}f(x)=\lim_{h\to 0}\frac{f\left(x+hx^{1-\alpha}\right)-f(x)}{h}\,. \tag{1.1}$$

Many analytical and numerical methods for accurate and approximate solutions have been developed by several researchers over the last few years, such as the Tanh method [5], Reliable methods [6], Exponential rational function method [7], Kudryashov method [8], Simplest equation method (SEM) [9], Single conformable Laplace transform method (CLT) [10], single conformable Laplace transform method (CLT) [10], conformable double Laplace transform (CDLT) [11,12], conformable triple Laplace transform and Sumudu transform [34,35], double Shehu transform [13], and some critical analyses about conformable fractional [20].

Watugala [14], implemented a new integral transformation at the beginning of the 1990s, called the Sumudu transformation (ST), which derived from the classical Fourier Transform, and applied it to obtain the solution of many problems in real life science and engineering. Infect, it is proved to be an efficient method for solving physical problems because of its unit and scale preserving properties. For more about (ST) see [15,16], authors have studied about properties of Sumudu transform and [17,18] application of Sumudu transform (ST) and Laplace transform (LT).

The transformation of Sumudu is defined in the set of functions [33],

$$\mathcal{A}=\{f(t)\mid \exists\, K,\ \tau_1,\tau_2>0,\ |f(t)|<Ke^{\frac{|t|}{\tau_j}},\ \text{if } t\epsilon(-1)^j\times[0,\infty)\}. \tag{1.2}$$

By the following formula,

$$F(u)=S[f(t)]=\int_0^{\infty}f(ut)e^{-t}\mathrm{d}u,\qquad u\in(-\tau_1,\tau_2). \tag{1.3}$$

The authors in [19,21] recently introduced the single and double conformable Sumudu transform (CST) in 2019-20. Ibrahim et.al [36-38], have explored operator for symmetric conformable fractional derivative of complex variable and on quantum hybrid fractional conformable differential in a complex domain, in [39,40], on subclasses of analytic functions based on a quantum symmetric and the generalized wave dynamical equations based on time space symmetric differential equation operator, respectively, and Moreover, in [41], some fixed-point theorems for almost weak contraction in S- metric space via conformable fractional operator. In order to solve linear fractional partial differential equations in the conformable fractional derivative sense, we implement the conformable Triple Sumudu transform (CDST) due to the certain benefits of Sumudu transformation