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Implicit Finite Difference Scheme for Singularly Perturbed Burger-Huxley Equations

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Abstract. In this paper, an implicit finite difference scheme is presented to solve one dimensional unsteady singularly perturbed Burger-Huxley equation. The quadratically convergent quasilinearization technique is used to linearize the nonlinear term of the equation. The innovative significance of this paper is the procedure to consider initial guesses in order to start the quasilinearization technique. This basic initial guessing causes to produce a more accurate solutions with the small iteration number for the problem under consideration. The derivatives are replaced by finite difference approximation, then we obtain the two-level time direction and the three-term recurrence relation in the spatial direction. The convergence analysis of the proposed method has been established. Numerical experiments were conducted to support the theoretical results. Further, the result shows that the proposed method gives a more accurate solution with a higher rate of convergence than some existing methods.

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1 Introduction

Consider one-dimensional unsteady singularly perturbed Burger-Huxley equation of the form:

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$$\begin{cases} \mathcal{L}_{x,\varepsilon}u(x,t) \cong -\varepsilon\dfrac{\partial^2 u}{\partial x^2} + \alpha u\dfrac{\partial u}{\partial x} + \dfrac{\partial u}{\partial t} - \beta(1-u)(u-\gamma)u = 0, & \forall (x,t)\in\mathcal{D}, \\ u(x,0) = u_0(x), & x\in\overline{\Omega} = [0,1], \\ u(0,t) = s_0(t), & t\in(0,T], \\ u(1,t) = s_1(t), & t\in(0,T], \end{cases} \tag{1.1}$$

where $0<\varepsilon\ll 1$ is a perturbation parameter. The solution domain $\mathcal{D}=(0,1)\times(0,T]$, and $\alpha\geq 1,\beta\geq 0,\gamma\in(0,1)$ are given constants. Such equation describes the interaction between convection, diffusion and reaction processes [1]. Burger-Huxley equation describes numerous fascinating phenomena such as busting oscillation [2], interspike [3], population genetics [4], bifurcation and chaos [5]. In the former few decades, several analytical methods were suggested to solve the Burger-Huxley equation. For example, by using Hirota method, Satsuma [6] obtained an exact solitary solution for this equation. Wang and other researchers in [7] built an exact solitary wave solution of the generalized Burger-Huxley equation. In [8], Wazwaz constructed some travelling wave solutions for the generalized forms of Burgers, Burgers-KdV and Burger-Huxley equation by using the standard tanh-function technique. The most innovative and robust numerical methods have been conducted on several families of singularly perturbed parabolic problems like in, [9–13].

Recently, several researches give attention to solve Burger-Huxley equation by the numerical methods such as different classes of domian decomposition method [14–17], variational iteration method [18], finite difference scheme [19, 20], different kinds of spectral methods [21, 22], family of collocation method [23, 24], computational meshless method [25]. However, the nature of the solution of singularly perturbed Burger-Huxley problem exhibits boundary layer. Because of the occurrence of this layer, the above mentioned methods are in question and known to be insufficient to estimate the accurate solution.

Consequently, to obtain uniformly convergent method, Kaushik and Sharma [26] establishes on nonuniform mesh for solving Eq. (1.1). Gupta and Kadalbajoo [27] assembled a numerical scheme that contains of implicit-Euler method which is first order uniformly accurate to discretize in temporal direction on a uniform mesh and a monotone hybrid finite difference operator to discretize the spatial direction with a piecewise uniform Shishkin mesh.

Research has been conducted to improve on the performance of Shishkin meshes while recalling some of their simplicity. The use of a piecewise uniform mesh with choosing transition parameter. The extendibility of the methods using meshes of Shishkin type to higher dimensional problem clarifies why people are interested in using them. Another advantage of Shishkin meshes over other types of meshes, is the convenience to handle complicated higher-order methods. Thus, in this paper, we aim at formulating higher-order fitted mesh of Shishkin type to solve the singularly perturbed Burger-Huxley equations.