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DATA-DRIVEN TIGHT FRAME CONSTRUCTION FOR IMPULSIVE NOISE REMOVAL*

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Abstract

The method of data-driven tight frame has been shown very useful in image restoration problems. We consider in this paper extending this important technique, by incorporating L_1 data fidelity into the original data-driven model, for removing impulsive noise which is a very common and basic type of noise in image data. The model contains three variables and can be solved through an efficient iterative alternating minimization algorithm in patch implementation, where the tight frame is dynamically updated. It constructs a tight frame system from the input corrupted image adaptively, and then removes impulsive noise by the derived system. We also show that the sequence generated by our algorithm converges globally to a stationary point of the optimization model. Numerical experiments and comparisons demonstrate that our approach performs well for various kinds of images. This benefits from its data-driven nature and the learned tight frames from input images capture richer image structures adaptively.

Mathematics subject classification: 68U10, 94A08.
Key words: Tight frame, Impulsive noise, Sparse approximation, Data-driven, Convergence analysis.

1. Introduction

Sparse approximation and representation are very powerful for many signal processing tasks, such as signal compression and image restoration. Sparse approximation models an image as a linear combination of a small number of elements of some system. Such system can be either a basis or an over-complete system, among which the wavelet tight frame [17] has been very successfully used in image restoration [8, 15, 18, 26]. Wavelet tight frames are able to approximate piecewise smooth signals efficiently with only few non-zero wavelet coefficients. Computationally, tight frames benefit from their efficient decomposition and reconstruction schemes. Numerous kinds of tight frames, including curvelets [12], ridgelts [11], framelets [17,36] and many others have been proposed for signal and image sparse representation. However, one certain tight frame cannot always perform well for all kinds of images. Naturally, it is better to design specific tight frame representation for a given image. The tight frame learned from the given image may achieve better performance for sparse approximation. Due to this nature, this kind of techniques are adaptive to input image data and hence work well for various types of images.

Cai et al. proposed in [9] a variational model under this idea, with an L_2-norm fidelity term and an L_0-norm regularization term. It is the first paper to propose data-driven tight

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frame model for image restoration, which shows comparable performance and runs much faster than general dictionary learning methods like K-SVD [21]. Bao et al. showed the sub-sequence convergence of the iterative algorithm in [3], and gave a new globally convergent algorithm. Soon later, this data-driven tight frame model has been improved and applied to various problems [13, 22, 27, 32, 33, 37, 42, 44]. However, as far as we know, these data-driven models mainly use L_2 fidelity, which is not suitable for the basic and significant impulsive noise removal problems.

Impulsive noise is usually divided into salt-and-pepper noise and random-valued noise. It is often generated by electromagnetic interference as well as faults and defects in communication system. It may also occur when the electrical switches and relays in the communication system change state [7]. Unlike the additive Gaussian noise, which affects all pixels of an image, salt-and-pepper noise (or random-valued noise) corrupts a portion of the pixels with minimal or maximal intensities (or random-valued intensities) while keeping remaining pixels unchanged. Hence, additive Gaussian noise removal algorithms are not suitable for impulsive noise removal.

Many impulsive noise removal algorithms have been proposed in recent decades, most of which can be categorized into nonlinear digital filter based methods [14, 19, 24, 25, 39] and variational regularization based approaches [2, 23, 29, 31, 40, 41, 43]. Nonlinear digital filter based methods improve median filter through weighting and adaptive techniques. Variational regularization approaches use total variation and some fidelity terms considering the noise statistics. These methods can achieve good performance in most cases. However, nonlinear digital filters are less capable of distinguishing noisy pixels from non-noisy pixels in edges or textured regions. Variational regularization term may introduce undesirable stair-casing effects. Considering the great successes of data-driven tight frame techniques in denoising, in this paper we are interested in extending [9] for impulsive noise removal. This yields an L_0 balanced model [9] with L_1 data fidelity, which is convenient for optimization and implementation. We will give an efficient alternating minimization algorithm to optimize the objective function and prove its global convergence. The iterative algorithm constructs discrete tight frames adapted to input images, which are then applied to the noise removal step. We mention that, a very recent paper [28], although considered data-driven tight frames for mixed Gaussian and impulsive noise removal, actually did not formulate an optimization model for data-driven tight frame construction in the presence of impulsive noise. Instead, their method uses directly the data-driven tight frame construction in [9] (with L_2 fidelity) as an independent package combined with an analysis approach [10] for denoising. This combination, if iteratively performed, yields a complicated two-loop procedure.

The rest of the paper is organized as follows. In Section 2, we briefly review the concept of tight frame and the existing data-driven tight frame construction. We propose our optimization model for impulsive noise removal in Section 3. It is implemented in patch space and solved by a proximal alternating minimization procedure. In Section 4, we show that the iterates of the proposed algorithm converge globally to a stationary point of the minimization model. Some numerical experiments are provided in Section 5. We conclude the paper in Section 6.

2. A Brief Review of Tight Frame and Data-driven Tight Frame Construction

In this section, we briefly introduce the tight frames and the data-driven tight frames construction proposed in [9]. Interested readers are referred to [10, 38] for more details.

Suppose that $\mathcal{H}$ is a Hilbert space. Let $\langle\cdot,\cdot\rangle$ and $\|\cdot\|$ denote the standard inner product, a