CSIAM Trans. Appl. Math.
doi: 10.4208/csiam-am.SO-2021-0010

Vol. **2**, No. 4, pp. 724-747
December 2021

A Saturation-Component Based Fuzzy Mumford-Shah Model for Color Image Segmentation

Wei Wang[1,*], Caifei Li[1] and Michael K. Ng[2]

[1] *School of Mathematical Sciences, Tongji University, Shanghai, P.R. China.*
[2] *Department of Mathematics, The University of Hong Kong, Pokfulam, Hong Kong.*

Received 1 February 2021; Accepted 18 May 2021

Abstract. In this paper, we propose and develop a novel saturation component based fuzzy Mumford-Shah model for color image segmentation. The main feature of this model is that we determine different segments by using the saturation component in hue, saturation, and value (HSV) color space instead of the original red, green and blue (RGB) color space. The proposed model is formulated for multiphase segmentation of color images with the assumption that a piecewise smooth function is approximated by the product of a piecewise constant function and a smooth function. The piecewise constant function and the smooth function are used to represent different segments and to estimate the bias field respectively in the color image. The approximation is calculated based on the saturation component which is particularly useful to distinguish edges and capture the inherent correlation among red, green and blue channels in color images. Experimental results are presented to demonstrate that the segmentation performance of the proposed model is much better than existing color image segmentation methods.

AMS subject classifications: 68U10, 65K10, 65J22, 90C25

Key words: Image segmentation, saturation, data fitting, energy minimization, iterative algorithm.

1 Introduction

The purpose of image segmentation is to partition an image into different regions according to their similar characteristics (such as intensities, textures, and colors). An overview of different segmentation methods is given in [1]. In the following discussion, we assume that $\Omega \subset R^2$ is a bounded open connected set, and $U: \Omega \rightarrow R^d$ is a given multi-channel image. For example, $d=1$ for gray-scale image, $d=3$ for RGB color image, $d>3$ for many

*Corresponding author. *Email addresses:* `wangw@tongji.edu.cn` (W. Wang), `1830957@tongji.edu.cn` (C. Li), `mng@maths.hku.hk` (M. K. Ng)

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cases such as hyperspectral image [2] and medical image [3]. Here we focus on color image ($d=3$).

Variational methods have been widely used for image segmentation. Well-known variational approaches include Mumford-Shah model [4, 5], region competition [6], geodesic active contour [7], geodesic active region [8], etc. In [4,5], Mumford and Shah presented a variational minimization problem which aimed to find an optimal piecewise smooth function to approximate the original gray image ($d=1$). More precisely, this problem was formulated as follows,

$$E_{MS}(b,\Gamma)=\lambda_1\int_\Omega (U-b)^2\,dx+\lambda_2\int_{\Omega-\Gamma}|\nabla b|^2\,dx+|\Gamma|, \tag{1.1}$$

where $|\Gamma|$ represents the length of Γ. b is an approximation of U, and it is smooth on each Ω_i which is a segment of the partition. λ_1 and λ_2 are two positive constants which are used to balance three terms. The Chan-Vese model [9] gave a simplification of objective function (1.1) where Γ partitioned the image domain into several segments with constant intensities. We note that the Mumford-Shah model is a nonconvex problem, and it is plagued with local minimizers. Therefore, it can be sensitive to initialization. Therefore, convex relaxation approaches [10–13], graph cut method [14], convex variant plus thresholding method [15–17] and fuzzy membership functions [18] were proposed to handle this problem.

In fuzzy segmentation method [10,12,19–21], each image pixel is assumed to be associated in some regions. The associated connection depends on fuzzy membership functions valued in [0,1]. On the other hand, bias field (intensity inhomogeneity) occurs in real images with different modalities, e.g., x-ray radiography/tomography and magnetic resonance (MR) images. It also occurs in natural images which is usually related with nonuniform illumination. In the literature, several approaches [22–25] have been proposed with bias correction. Li et al. [24] proposed a variational model for medical image segmentation and bias correction based on level set method. They defined an objective function for local intensities, and then integrated it to formulate a functional in terms of the level set function and the bias field. However, the numerical scheme for the level set framework is computationally expensive. In [25], Li et al. assumed that a piecewise smooth function is approximated by a product of a piecewise constant function and a smooth function. The smooth function is proposed to estimate the bias field. Their model is given as

$$U(x)=g(x)c_i+n(x), \quad x\in\Omega_i.$$

This model has been widely applied in handling intensity inhomogeneity [23,24,26]. The smooth function g represents the bias field, and it is assumed to be valued around 1. c_i is a constant and n represents the noise. The fuzzy Mumford-Shah (FMS) functional was