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Asymptotic Stability of Shock Wave for the Outflow Problem Governed by the One-Dimensional Radiative Euler Equations

Lili Fan[1], Lizhi Ruan[2] and Wei Xiang[3,*]

[1] *School of Mathematics and Computer Science, Wuhan Polytechnic University, Wuhan 430023, China.*
[2] *The Hubei Key Laboratory of Mathematical Physics, School of Mathematics and Statistics, Central China Normal University, Wuhan 430079, China.*
[3] *Department of Mathematics, City University of Hong Kong, Tat Chee Avenue, Kowloon, Hong Kong, China.*

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Abstract. This paper is devoted to the study of the asymptotic stability of the shock wave of the outflow problem governed by the one-dimensional radiative Euler equations, which are a fundamental system in the radiative hydrodynamics with many practical applications in astrophysical and nuclear phenomena. The outflow problem means that the flow velocity on the boundary is negative. Comparing with our previous work on the asymptotic stability of the rarefaction wave of the outflow problem for the radiative Euler equations in [6], two points should be pointed out. On one hand, boundary condition on velocity is considered instead of boundary condition on temperature, which induces a perfect boundary condition on anti-derivative perturbations so that boundary estimates on perturbed unknowns are trickily and smoothly established. On the other hand, the rarefaction wave is an expansive wave, while the shock wave is a compressive wave. So we need take good advantages of properties of the shock wave instead. Our investigation on the outflow problem provides a good understanding on the radiative effect and boundary effect in the setting of shock wave.

*Corresponding author. *Email addresses:* `rlz@mail.ccnu.edu.cn` (L. Ruan), `weixiang@cityu.edu.hk` (W. Xiang), `fll810@live.cn` (L. Fan)

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1 Introduction

In this paper, we will continue to study the outflow problem governed by the one-dimensional radiative Euler equations, which is the second one of our series of papers on such kind of outflow problem, actually, the first one on the asymptotic stability of the shock wave for the radiative Euler equations with a boundary. The radiative Euler equations are a fundamental system to describe the motion of the compressible gas with the radiative heat transfer phenomena, which has many applications in astrophysics and nuclear explosions. Mathematically, the one-dimensional radiative Euler equations in the Eulerian coordinates can be modelled as a hyperbolic-elliptic coupled system of the following form:

$$\begin{cases} \rho_t+(\rho u)_x=0, \\ (\rho u)_t+\left(\rho u^2+p\right)_x=0, \\ \left\{\rho\left(e+\dfrac{u^2}{2}\right)\right\}_t+\left\{\rho u\left(e+\dfrac{u^2}{2}\right)+pu\right\}_x+q_x=0, \\ -q_{xx}+aq+b\left(\theta^4\right)_x=0, \end{cases} \tag{1.1}$$

where ρ, u, p, e and θ are respectively the density, velocity, pressure, internal energy and absolute temperature of the gas, and q is the radiative heat flux. Positive constants a and b depend only on the gas itself. Like the classic compressible Euler equations, the first three equations in (1.1) stand for the conservation of the mass, momentum and energy respectively. The fourth equation in (1.1) is related to the radiative heat transfer phenomenon, and one can refer [1,9,23,28,39,44] for more details. System (1.1) can also be derived by the non-relativistic limit (speed of light tending to $+\infty$) from a hyperbolic-kinetic system, and rigorous mathematical derivation can be found in [15].

Precisely speaking, we will investigate the initial-boundary value problem of system (1.1) in the half space $\{(x,t)|0\leq x,t<\infty\}$ with the initial data

$$(\rho,u,\theta)(x,0)=(\rho_0,u_0,\theta_0)(x) \quad \text{for} \quad x\geq 0, \tag{1.2}$$

satisfying

$$\inf_{x\in[0,+\infty)}(\rho_0,\theta_0)(x)>0 \tag{1.3}$$