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An Efficient Variational Model for Multiplicative Noise Removal

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Abstract. In this paper, an efficient variational model for multiplicative noise removal is proposed. By using a MAP estimator, Aubert and Aujol [SIAM J. Appl. Math., 68(2008), pp. 925-946] derived a nonconvex cost functional. With logarithmic transformation, we transform the image into a logarithmic domain which makes the fidelity convex in the transform domain. Considering the TV regularization term in logarithmic domain may cause oversmoothness numerically, we propose the TV regularization directly in the original image domain, which preserves more details of images. An alternative minimization algorithm is applied to solve the optimization problem. The z-subproblem can be solved by a closed formula, which makes the method very efficient. The convergence of the algorithm is discussed. The numerical experiments show the efficiency of the proposed model and algorithm.

AMS subject classifications: 65M10, 78A48

Key words: Multiplicative noise, variational model, alternating direction minimization.

1. Introduction

Image recovery encompasses the large body of inverse problems, in which a multidimensional signal u is inferred from the observation data f, consisting of signals physically or mathematically related to it. The objective is to recover the original image from the observation of a contaminated image. The original image can be degraded by different mathematical methods. The most famous variational model for additive noise removal is Rudin-Osher-Fatemi (ROF) model [24]. Many other variational methods are also proposed, for instance [9, 11, 13, 16, 21, 27]. The ROF model introduced

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total variational minimization to image processing, which can preserve the edges well. The additive noise model is generated from the model $f = u + \eta$, where f, u and η are observed image, true image and the additive noise, respectively. The ROF model defined the solution as follows:

$$u = \underset{u \in BV(\Omega)}{\operatorname{argmin}} |u|_{BV} + \frac{\lambda}{2}\|f - u\|_{L^2}^2$$

for a regularization parameter $\lambda > 0$, where $BV(\Omega)$ denotes the space of function with bounded variation on Ω, equipped with the BV seminorm which is formally given by

$$|u|_{BV} = \int_\omega |\nabla u|,$$

also noted as the total variation (TV) of u.

Multiplicative noise, also known as speckle noise, has not been discussed thoroughly when compared with additive noise. Multiplicative noise occurs in many areas, such as magnetic field inhomogeneity in MRI [2], ultrasound images [17], synthetic aperture radar (SAR) images [19], etc. We consider the problem of recovering the original image u, which is corrupted by multiplicative noise η. i.e., $f = u\eta$ with assumption $f > 0$. One often assume that the noise η follows Gamma distribution, which commonly occurs in SAR. The probability density function of η is denoted as P_η,

$$P_\eta(x, \theta, K) = \frac{1}{\theta^K \Gamma(K)} x^{K-1} e^{-\frac{x}{\theta}}, \tag{1.1}$$

where Γ is Gamma function, and θ and K denote the scale and shape, respectively. The mean of η is $K\theta$ and the variance of η is $K\theta^2$.

The total variation approach to multiplicative noise model was proposed by Rudin *et al.* [23], in which the multiplicative noise is assumed to Gaussian distribution. Although [23] can restore images well, the Gaussian multiplicative noise is not common in real applications. For the Gamma distributed multiplicative noise, Aubert and Aujol [1] proposed a variational model based on the maximum a posteriori (MAP) estimator as follows, which is referred to as AA model

$$\inf_{u \in S(\Omega)} \int_\Omega \left(\log u + \frac{f}{u}\right) dx + \lambda \int_\Omega |Du|. \tag{1.2}$$

The second term is the TV regularization term and λ is regularization parameter to trade-off. Although the fidelity term in (1.2) is not convex, they also give the existence of minimizer and proved the uniqueness with a sufficient condition. The nonconvex fidelity raises the difficult to achieve the global optimal, and the numerical result often dependent on the initial guess. Based on AA model, spatially varying regularization parameter is discussed in [20] to get more texture details. By the logarithmic transformation, $f = u\eta$ turns to $\log f = \log u + \log \eta$, which can be treated as additive noise. Relaxed inverse scale space (RISS) flow is used to solve u in [25] and we denote it