On a New Type of Chemotaxis Model with Acceleration

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Abstract. In this paper, we consider a new type of chemotaxis model with acceleration, which assumes that the advective acceleration, instead of velocity in the classical chemotaxis model, of species is proportional to the chemical signal concentration gradient. This new model has an additional equation governing the velocity field with more delicate boundary conditions. We show that this new type of chemotaxis model with acceleration precludes the blow-up and admits globally bounded solutions in two and three dimensions, in contrast to the classical chemotaxis models which may have blow-up in three dimensions or in two dimensions with a critical mass. Moreover, we numerically illustrate that this new type of chemotaxis model can typically generate aggregation patterns. Some open questions are discussed for further studies.

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1   Introduction

The reaction-diffusion systems are widely used to describe various biological processes such as propagation of genetics [5,22], sundry pattern formation [9,19,24],

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ecological invasions [2,4,12], tumour growth [3], wound healing [37], and so on. In these models, random diffusion was used as the only dispersal strategy [17,38], which however cannot explain some more complex ecological processes involving the rational movement (directed motion of individuals dispersing so as to increase the chance of survival [8]), nor accurately reflect the non-Brownian motion of individuals [34]. For example, for the Lotka-Volterra type predator-prey system, if the movement is assumed to be random diffusion then no spatial patterns shall be observed [15,44], which can not support the spatio-temporal heterogeneity patterns observed in the field experiment [17,41]. One common way to model the rational movement is the use of taxis mechanism, such as chemotaxis [18] or preytaxis [14,17]. A well-known classical chemotaxis model was the following so-called minimal model [28]:

$$
\begin{cases}
u_t = \Delta u - \nabla \cdot (u \nabla v), \\
\tau v_t = \Delta v - v + u,
\end{cases}
\tag{1.1}
$$

where u and v denote the cell density and the concentration of the chemical signal emitted from cells, respectively, $\tau \in \{0,1\}$. The chemotaxis term $\nabla \cdot (u \nabla v)$ accounts for the rational movement. The preytaxis system also has the similar rational movement structure (cf. [14,17]). In these models, the advective velocity of species is assumed to be directly proportional to the concentration gradient of signals/resources, and the details of how species respond to signal concentration gradient are not reflected. On the other hand, there are many observations of the dependence of individual acceleration on the signal gradient, such as acceleration vectors of individuals in fish schools (cf. [30]) and in swarms of flying insects (cf. [32]) are directed towards the centroid of such dynamically stable formations, the moving flea-beetles modify their acceleration in response to food patch quality (cf. [16]) and individual fish in schools adjust their variation of velocity according to the difference between ambient and preferred temperatures (cf. [6]). In these observations, the directed movement of individuals is not determined by the velocity itself but by the velocity variation (i.e., acceleration) which is proportional to the resource gradient. This type of rational movement with advective velocity accelerating with respect to resource gradient has been mathematically modelled in the works [1,35] which found that the model can generate the spatio-temporal patterns consistent with the experimental observations while the conventional rational movement modelled by taxis directly can not do (cf. [15,44]). Such kind of models with acceleration incorporate more detailed responses of migrants to the signal instead of simply advecting to the signal/resource gradient. The purpose of this paper is to apply this type of rational movement to chemotaxis and explore what will be different from the classical chemotaxis model (1.1).