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New Splitting Algorithms for Multiplicative Noise Removal Based on Aubert-Aujol Model

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Abstract. In this paper, we propose new algorithms for multiplicative noise removal based on the Aubert-Aujol (AA) model. By introducing a constraint from the forward model with an auxiliary variable for the noise, the NEMA (short for Noise Estimate based Multiplicative noise removal by alternating direction method of multipliers (ADMM)) is firstly given. To further reduce the computational cost, an additional proximal term is considered for the subproblem with regard to the original variable, the NEMA_f (short for a variant of NEMA with fully splitting form) is further proposed. We conduct numerous experiments to show the convergence and performance of the proposed algorithms. Namely, the restoration results by the proposed algorithms are better in terms of SNRs for image deblurring than other compared methods including two popular algorithms for AA model and three algorithms of its convex variants.

AMS subject classifications: 15A29, 65K10, 68U10, 94A08

Key words: Alternating direction method of multipliers, image denoising and deblurring, multiplicative noise, total variation.

1. Introduction

Image restoration is of great significance in image processing, as the noise and blur are ubiquitous in image acquisition and transmission. In most existing works, additive noise models [7,24,27] have been widely studied, aiming to recover the original image u from the observed image f corrupted by the additive noise v. As one of the typical non-Gaussian noise, the multiplicative noise [2, 10, 19, 21, 36, 40, 44] have also been widely studied. It has severe pollution on the clean images, such degradation can be formulated as the clean image u blurred by a known linear operator $A \in \mathbb{R}^{N\times N}$ and

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further corrupted by the multiplicative noise η, i.e.

$$f = (Au) \circ \eta, \tag{1.1}$$

where f, u, and $\eta \in \mathbb{R}^N$ all denote the $\sqrt{N} \times \sqrt{N}$ image rewritten as a vector in an alphabetical order, N denotes the number of pixels, $\circ$ denotes the Hadarmard product of two vectors and η denotes the noise following independent and identically distributed Gamma distribution on each pixel with mean 1. A typical example of multiplicative noise is the speckle noise in synthetic aperture radar images [29]. In addition, the multiplicative noise also appears in ultrasound imaging [39], laser [35], and other coherent image systems [3, 4, 38].

Researchers have developed various denoising algorithms for such noise, including total variation (TV) regularization based methods [8], non-local low rank based methods [45], diffusion based methods [46, 49], non-local patch-based methods [9, 30], and deep learning based methods [41]. Among them, TV regularized model is one of the most widely used methods due to its edge-preserving and scale-dependent properties [8]. Rudin *et al.* [33] firstly applied the TV based method to multiplicative noise removal. Aubert and Aujol [1] derived a non-convex variational model (AA model) via the maximum a posteriori (MAP) estimation for Gamma noise. Huang *et al.* [16] established a strictly convex model (HNW model) by introducing a new variable and a new fitting term. By utilizing the statistical properties of the multiplicative Gamma noise, Dong and Zeng [10] proposed a convex model (DZ model) by adding a quadratic penalty term. Other convex approach was proposed by Zhao *et al.* [48] (ZWN model), where they decoupled variables by transforming the form of the multiplicative noise equation. Besides, Li and Lou [20] proposed the AA-DCA algorithm by reformulating the AA model into the difference of convex (DC) functions programming. Lu *et al.* [25] proposed a variational convex model by setting a new term to replace the quadratic penalty in DZ, and transforming it into a exp-model according to the fact that pointed in [16, 36] the exponent-like models usually perform better than logarithm-like counterparts in terms of the quality of denoised images.

In many applications, restoration results demonstrate that non-convex methods outperform convex one, e.g. image denoising [7, 23, 26–28] and image segmentation [31, 37]. Hence in this paper, we are interested in designing more efficient algorithms for the original nonconvex AA model, in order to achieve optimal capability of the multiplicative noise removal. By introducing a constraint from the forward model with an auxiliary variable for the noise, we propose an efficient splitting algorithm based on alternating direction method of multipliers (ADMM) [5, 11, 13, 14, 43]. In order to avoid extra computational cost of inner loop due to the existence of the TV term, we further present a fully splitting form by introducing constraint of the gradient and additional proximal terms. The main contribution of this paper is given below.

- We design an operator splitting algorithm for Noise Estimate based Multiplicative noise removal by ADMM (NEMA) for the original AA model by introducing a constraint with an auxiliary variable for the noise such that the subproblem of the noise variable has a closed-form solution.