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Exact Boundary Controllability of Fifth-order KdV Equation Posed on the Periodic Domain

YANG Shuning[1] and ZHAO Xiangqing[2,*]

[1] *School of Information and Engineering, Zhejiang Ocean University, Zhoushan 316022, China.*
[2] *Department of Mathmatics, Suqian University, Suqian 223800, China.*

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Abstract. In this paper, we show by Hilbert Uniqueness Method that the boundary value problem of fifth-order KdV equation

$$\begin{cases} y_t - y_{5x} = 0, \quad (x,t) \in (0,2\pi) \times (0,T), \\ y(t,2\pi) - y(t,0) = h_0(t), \\ y_x(t,2\pi) - y_x(t,0) = h_1(t), \\ y_{2x}(t,2\pi) - y_{2x}(t,0) = h_2(t), \\ y_{3x}(t,2\pi) - y_{3x}(t,0) = h_3(t), \\ y_{4x}(t,2\pi) - y_{4x}(t,0) = h_4(t), \end{cases}$$

(with boundary data as control inputs) is exact controllability.

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Key Words: Fifth-order KdV equation; Hilbert Uniqueness Method; exact controllability.

1 Introduction

In [1,2], the authors have studied the internal controllability of the fifth-order Korteweg-de Vries equation posed on a periodic

*Corresponding author. *Email addresses:* yangsn821@163.com (S. N. Yang), zhao-xiangqing@163.com (X. Q. Zhao)

$$\begin{cases} u_t+\alpha u_{5x}+\beta u_{3x}+\gamma u u_x=f(t,x), \quad (x,t)\in(0,2\pi)\times(0,T),\\ u(t,2\pi)-u(t,0)=0,\\ u_x(t,2\pi)-u_x(t,0)=0,\\ u_{2x}(t,2\pi)-u_{2x}(t,0)=0,\\ u_{3x}(t,2\pi)-u_{3x}(t,0)=0,\\ u_{4x}(t,2\pi)-u_{4x}(t,0)=0, \end{cases} \tag{1.1}$$

where the external forcing function $f=f(x,t)$ is considered as a control input and is assumed to be supported in a given open set $\omega\subset(0,2\pi)$. The main result is as follows:

Theorem A [Global controllability] *Let $R>0$ be given. There exists a $T>0$ such that for any u_0, $u_1\in H^s(\mathbb{T})$ $(s\geq 0)$ with $[u_0]=[u_1]([a]=\frac{1}{2\pi}\int_0^{2\pi}a(x)\mathrm{d}x)$ and*

$$\|u_0\|_{L^2(\mathbb{T})}\leq R, \qquad \|u_1\|_{L^2(\mathbb{T})}\leq R,$$

one can find a control input $h\in L^2(0,T;H^s(\mathbb{T}))$ such that the system (1.1) *admits a solution $u\in C(0,T;H^s(\mathbb{T}))$ satisfying*

$$u|_{t=0}=u_0, \qquad u|_{t=T}=u_1.$$

Naturally, we will ask the question that how about boundary controllability of the system. Specifically, is the following system

$$\begin{cases} y_t-y_{5x}=0, \quad (x,t)\in(0,2\pi)\times(0,T),\\ y(t,2\pi)-y(t,0)=h_0(t),\\ y_x(t,2\pi)-y_x(t,0)=h_1(t),\\ y_{2x}(t,2\pi)-y_{2x}(t,0)=h_2(t),\\ y_{3x}(t,2\pi)-y_{3x}(t,0)=h_3(t),\\ y_{4x}(t,2\pi)-y_{4x}(t,0)=h_4(t), \end{cases}$$

(where h_0,h_1,h_2,h_3,h_4 are considered as control inputs) exactly controllable? In this paper, we'll answer this question. Before stating the main result, we give the notation

$$H_p^k=\left\{u\in H^k(0,2\pi):u^{(j)}(0)=u^{(j)}(2\pi), \quad \text{for } 0\leq j\leq k-1\right\},$$

where $H^k(0,2\pi)$ denotes classical Sobolev space on the interval $(0,2\pi)$. It is easy to see

$$u\in H_p^k\Leftrightarrow\sum_{n\in\mathbb{Z}}\left(n^k|\hat{u}(n)|\right)^2<\infty,$$

and that the Sobolev norm