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Analysis and FDTD Simulation of a Perfectly Matched Layer for the Drude Metamaterial

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Abstract. In this paper, we are concerned about the stability analysis for a Perfectly Matched Layer (PML) recently developed by Bécache et al. [5] for simulating wave propagation in the Drude metamaterial. This PML is proved to be stable originally in [6] through a modal analysis. Here we establish its stability by the energy method. A FDTD scheme is developed and analyzed. Numerical simulations illustrate the stability of the PML model and its effectiveness in absorbing outgoing waves in the Drude medium.

AMS subject classifications: 65N30, 35L15, 78-08

Key words: Maxwell's equations, perfectly matched layer, FDTD method.

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1 Introduction

One of the challenges to simulate wave propagation in unbounded domains is how to construct effective artificial boundary conditions to absorb the outgoing waves without reflecting them back into the computational domains. A widely adopted technique is the so-called Perfectly Matched Layer (PML) proposed by Bérenger [7] in 1994 for solving the three-dimensional (3D) time-dependent Maxwell's equations. Since 1994, in addition to many PML models proposed and studied further for Maxwell's equations [1,8,10,11,28,35,36], the PML technique has also been extended

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to solve other wave propagation problems, such as acoustics and elastodynamics [2, 5, 16].

In late 1990s, the so-called negative index metamaterials (NIMs) was manufactured successfully [30, 32] and immediately became a very hot research topic as evidenced by numerous papers (cf. [37] and references therein) and books published on metamaterials (e.g., [13, 17, 24] and references therein). Due to the importance of numerical simulation for NIMs, many studies of PMLs in NIMs have been carried out (e.g., [12,15,31]). Cummer [14] first noticed that the classical PMLs fail in NIMs and proposed stable PMLs for the Drude metamaterial with $\omega_e=\omega_m$ (see (2.2) below). Stable PMLs were later extended to the general case $\omega_e\neq\omega_m$ in [15,31]. In [5], Bécache et al. presented a rigorous development of a stable PMLs for the Drude model for the general case $\omega_e\neq\omega_m$. The stability is proved in [6] through a modal analysis. Since the modal analysis is limited to the constant damping coefficients, one of our main goals of this paper is to make an effort in establishing a stability for the practical variable damping functions by using the energy method. To our best knowledge, this is the first energy stability established for this PML model.

Since the PML models (cf. [33, Ch. 7], [34], [24, Ch. 8] and references therein) are much more complicated than the corresponding Maxwell's equations, the stability analysis is quite challenging. For example, the stability for the classical Bérenger PML with variable damping functions is made possible through an equivalent form [4]. Furthermore, developing and analyzing effective numerical methods for solving the PML models is not trivial, and many researchers have made contributions in this direction (e.g., [3, 9, 19–23, 26, 27]). Though Bécache et al. [5] have presented Finite-Difference Time-Domain (FDTD) simulation for their developed metamaterial PML model, but no detail has been given for the FDTD scheme and its analysis. Hence, another major goal of our paper is to fill the gap by developing and analyzing a FDTD scheme for the metamaterial PML model proposed by Bécache et al. [5].

The rest of the paper is organized as follows. In Section 2, we first introduce the 2D metamaterial PML model proposed in [5], and then carry out its stability analysis. In Section 3, we propose a FDTD scheme for this PML model, and establish a discrete stability. Numerical results are presented in Section 4 to demonstrate the stability of this PML model and its effectiveness in absorbing outgoing waves. We conclude the paper in Section 5.

2 The 2-D metamaterial PML model

A general 2-D Transverse Electric (TEz) metamaterial PML model wth 16 unknowns was developed in Bécache et al. Here we focus on the popular $\omega_e=\omega_m$ case whose