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On Strong Deviation Theorems Concerning Arrays of Non-negative Integer-valued Random Variables

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Abstract. In this article, by using the notion of general relative entropy as a measure of dependence of random variables, a kind of strong deviation theorems for the averages of array of integer-valued random variables are established. At the end of this paper, we give the upper and lower bounds for general moving averages of the form.

AMS subject classifications: 60F15

Key words: Generating function, strong deviation theorem, general relative entropy.

1 Introduction

Since Clausis introduced entropy into thermodynamics in 1854. It had been extensively investigated in the literature. Shannon (1948) made use of it in Information Theory to measure the capacity of a communication device (see [8]). Kolmogorov (1958) carried it over to stochastic processes. Kullback (1967) generalized the idea of entropy called "discrimination". This form of information measure is now more commonly refereed to as relative entropy or cross entropy and it was better interpreted as a measure of similarity between random variables (see [3-4]). A recent active research that had made a good use of the idea of relative entropy is that of economics theory, for example, Blume and Easley (1992) considered a multi-stage economy, governed by an unknown stationary stochastic process. Each agent had a "misspecified" model. This meant that the agent was undecided between two possible models regarding the stochastic process. They showed that the logarithm of the likelihood ratio between a model's distribution and the true distribution, namely the relative entropy, were the critical parameter in determining whether learning occurs (see [1]). Lahrer et al. (2000) introduced the notion of relative entropy,

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which were a natural adaptation of the entropy of a stochastic process to the subjective set-up. They presented conditions, expressed in terms of relative entropy, which determined whether the agent would eventually learn to optimize and introduced the notion of pointwise merging and linked it to relative entropy (see [5]).

In recent year, Wang and Chen (2014) have studied the almost sure limiting behavior of the generalized delayed averages of random sequence by virtue of the notion of asymptotic delayed log-likelihood ratio and Laplace transformation (see [10]). Wang and Yang (2017) proposed a new concept of the generalized entropy density, and established a generalized entropy ergodic theorem for time inhomogeneous Markov chain.

Motivated by the work of Lahrer et al.(2000), Wang and Chen (2014) and Wang and Yang (2017), we introduce some new concepts of moving likelihood ratio and generalized relative entropy in this paper. The definition is a natural extension of the classical ones and have an intrinsic mathematical interest and is very natural. We try to connect the generalized relative entropy (See Equation (2.5) for mathematical definition) with the frequency of some alphabet in a fixed segment. The generalized relative entropy we introduced is defined in terms of the moving likelihood ratio of the reference measure and the truth. We restrict ourselves to random variables with countable state for sake of simplicity (see [5-7]).

The approach to proving the main results in this paper follows the line of the technique method presented by Wang and Yang (2017). The key issue of the proofing is first to construct non-negative random variables depending on a common parameter before applying the Borel-Cantelli lemma to obtaining some strong limit theorems (see [9]).

The rest of this paper is organized as follows. In Section 2, we first introduce some definitions, and present some preliminaries that form the basis for the main results. In Section 3, we present the main results and the proofs.

2 Preliminaries

Suppose two probability measures μ and $\widetilde{\mu}$ be defined on the same space $(\Omega,\mathcal{F})$ and $\{\xi_{ni},v_n\leqslant i\leqslant u_n\}_{n\in\mathbb{N}^+}$ be an array of non-negative integer-valued random variables (r.v.s for short). Assume that the joint distributions of $\{\xi_{ni},v_n\leqslant i\leqslant u_n\}_{n\in\mathbb{N}^+}$ under μ and $\widetilde{\mu}$ respectively, are

$$p_n(x_{nv_n},\cdots,x_{nu_n})=\mu(\xi_{nv_n}=x_{nv_n},\cdots,\xi_{nu_n}=x_{nu_n})>0,\ \ x_{ni}\in\mathbb{N},\ v_n\leqslant i\leqslant u_n,\ n\in\mathbb{N}^+, \quad (2.1)$$

and

$$q_n(x_{nv_n},\cdots,x_{nu_n})=\widetilde{\mu}(\xi_{nv_n}=x_{nv_n},\cdots,\xi_{nu_n}=x_{nu_n}),\ \ x_{ni}\in\mathbb{N},\ v_n\leqslant i\leqslant u_n,\ n\in\mathbb{N}^+, \quad (2.2)$$

where $\{(v_n,u_n):v_n<u_n\in\mathbb{Z},v_n>-\infty,u_n<+\infty\}$, $\mathbb{N}=\{0,1,2,\cdots\}$, $\mathbb{N}^+=\{1,2,\cdots\}$.

If $\{\xi_{ni},v_n\leqslant i\leqslant u_n\}_{n\in\mathbb{N}^+}$ is rowwise independent under measure $\widetilde{\mu}$, then there exists an array of distributions on $\mathbb{N}$

$$(p_{ni}(0),p_{ni}(1),\cdots),\quad v_n\leqslant i\leqslant u_n,\quad n\in\mathbb{N}^+, \quad (2.3)$$