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GLOBAL DYNAMICS OF A PREDATOR-PREY MODEL WITH PREY REFUGE AND DISEASE*†

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Abstract

In this paper, we study a predator-prey model with prey refuge and disease. We study the local asymptotic stability of the equilibriums of the system. Further, we show that the equilibria are globally asymptotically stable if the equilibria are locally asymptotically stable. Some examples are presented to verify our main results. Finally, we give a brief discussion.

Keywords predator-prey model; prey refuge; disease; stability

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1 Introduction

Predator-prey model is one of the basic models between different species in nature. These models have been studied extensively and many excellent results have been obtained (see [1, 2]). On the other hand, the effect of disease in ecological system is an important topic from mathematical as well as ecological point of view. After the work of Kermack-McKendrick [3] on SIRS (susceptible-infected-removed-susceptible) systems, many authors have investigated the dynamical behavior of epidemiological models. Chattopadhyay and Arino [4] proposed a predator-prey epidemiological model with disease spreading in prey. They assumed that the sound prey population grows according to a logistic law involving the whole prey population, and discussed the positivity, uniqueness, boundedness of the solutions and the existence of supercritical Hopf bifurcation. Haque et al [5] investigated a Lotka-Volterra type predator-prey model with a transmissible disease in the predator species. They aussumed that the sound and infected predators can hunt the prey and studied the stability of system.

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Sun and Yuan [6] proposed the following predator-prey model with disease in the predator

$$\begin{aligned} x' &= x(a - bx) - cxS, \\ S' &= exS - d_1S - \beta SI, \\ I' &= \beta SI - d_2I, \end{aligned} \tag{1.1}$$

where $x(t), S(t)$ and $I(t)$ represent the densities of the prey, susceptible (sound) predator and the infected predator population at time t, respectively. They assumed that there is a spread of disease in predator and only the susceptible predators have ability to capture prey. They investigated the boundedness of solution and global asymptotical stability of the equilibriums.

On the other hand, prey species makes use of refuges to decrease predation risk, here refuge means a places or situations where predation risk is somehow reduced. Ma et al [7] studied the following predator-prey model with prey refuges and a class of functional responses

$$\begin{aligned} X' &= rx\Big(1 - \frac{x}{K}\Big) - p\varphi(X - X_R)Y, \\ Y' &= (q\varphi(X - X_R) - d)Y, \end{aligned} \tag{1.2}$$

where the term $\varphi(X)$ represents the functional response of the predator population. They obtained the local asymptotical stability of equilibrium point and showed that the refuges used by prey can increase the equilibrium density of prey population but decrease that of predator. Ma et al. [8] further studied the influence of prey refuge and density dependent of predator species on the traditional Lotka-Volterra model. Huang, Chen and Li [9] studied the influence of prey refuge on a predator-prey model with Holling type III response function. In [10], a global analysis of a Holling type II predator-prey model with a constant prey refuge was presented. Ma et al [11] and Chen, Chen and Wang [12] studied a Lotka-Volterra predator-prey model incorporating a prey refuge and predator mutual interference. For more details in this direction, please see [13, 14].

However, there are still seldom scholars investigating the predator-prey model with prey refuge and disease in predator. More precisely, we study the global stability of the following model

$$\begin{aligned} x' &= rx\left(1 - \frac{x}{K}\right) - c(1 - m)xS, \\ S' &= e(1 - m)xS - d_1S - \beta SI, \\ I' &= \beta SI - d_2I, \end{aligned} \tag{1.3}$$

where $x(t)$, $S(t)$ and $I(t)$ represent the densities of the prey, susceptible (sound) predator and the infected predator population at time t, respectively with initial