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OSCILLATION CRITERIA FOR EVEN ORDER DELAY DIFFERENTIAL EQUATIONS WITH NONLINEAR TERM*

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Abstract

By Riccati transformation, we establish some new oscillation criteria for a class of even order delay differential equations with nonlinear term. In some sense, the results obtained extend some known results in the literature.

Keywords oscillation; delay; nonlinear term; even order; Riccati transformation

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1 Introduction

In this paper, we study the problem of oscillation for a class of even order neutral delay differential equations

$$(r(t)[x(t)+c(t,x(\tau(t)))]^{(n-1)})'+q(t)f(x(\sigma(t)))=0, \quad t\geq t_0. \tag{1.1}$$

Throughout, we suppose that the functions and parameters in (1.1) satisfy the following conditions:

(A1) $r\in C([t_0,\infty),\mathbf{R}^+)$, $\int_{t_0}^{\infty}\frac{\mathrm{d}t}{r(t)}=\infty$, n is even;

(A2) $\tau\in C^1([t_0,\infty),\mathbf{R})$, $\sigma\in C([t_0,\infty),\mathbf{R})$, $\tau(t)\leq t$, $\lim\limits_{t\to\infty}\tau(t)=\lim\limits_{t\to\infty}\sigma(t)=\infty$, $\tau\circ\sigma=\sigma\circ\tau$;

(A3) $q\in C([t_0,\infty),\mathbf{R}^+)$;

(A4) $c\in C([t_0,\infty)\times\mathbf{R},\mathbf{R})$, there exist a function $p\in C([t_0,\infty),\mathbf{R}^+)$ and a positive constant p_0 such that

$$0\leq\frac{c(t,u)}{u}\leq p(t)\leq p_0<\infty \quad \text{for } u\neq 0;$$

(A5) $f\in C(\mathbf{R},\mathbf{R})$ and there exists a positive constant α such that

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$$\frac{f(y)}{y} \geq \alpha > 0, \quad \text{for } y \neq 0;$$

Neutral differential equations have numerous applications in natural science and technology. For instance, they are frequently used to study the distributed networks containing lossless transmission lines, see Hale [1].

In the last decades, there are many studies that have been made on the oscillatory behavior of solutions of differential equations [2-6] and neutral delay differential equations [7-20].

For instance, Grammatikopoulos et al. [9] examined the oscillation of second-order neutral delay differential equations

$$[x(t) + p(t)x(t-\tau)]'' + q(t)x(t-\sigma) = 0, \quad t \geq t_0, \tag{1.2}$$

where $0 \leq p(t) < 1$.

Liu and Bai [12] investigated the second-order neutral differential equations

$$[r(t)|Z'(t)|^{\alpha-1}Z'(t)]' + q(t)|y(\sigma(t)|^{\alpha-1}y(\sigma(t)) = 0, \quad t \geq t_0, \tag{1.3}$$

where $Z(t) = y(t) + p(t)y(\tau(t)),\ 0 \leq p(t) < 1$.

Meng and Xu [13] studied the oscillation of even-order neutral delay differential equations

$$[r(t)|(x(t)+p(t)x(t-\tau))^{n-1}|^{\alpha-1}(x(t)+p(t)x(t-\tau))^{n-1}]'+q(t)f(x(\sigma(t))) = 0, \quad t \geq t_0, \tag{1.4}$$

where $0 \leq p(t) < 1$.

Ye and Xu [16] considered the second-order quasilinear neutral delay differential equations

$$[r(t)\Psi(x(t))|Z'(t)|^{\alpha-1}Z'(t)]' + q(t)f(x(\sigma(t))) = 0, \quad t \geq t_0, \tag{1.5}$$

where $Z(t) = x(t) + p(t)x(\tau(t)),\ 0 \leq p(t) < 1$.

Zafer [17] discussed the second-order neutral delay differential equations

$$[x(t) + p(t)x(\tau(t))]^{(n)} + f(t, x(t), x(\sigma(t))) = 0, \quad t \geq t_0, \tag{1.6}$$

where $0 \leq p(t) < 1$.

Zhang et al. [18] considered the oscillation of even-order nonlinear neutral delay differential equations

$$[x(t) + p(t)x(\tau(t))]^{(n)} + q(t)f(x(\sigma(t))) = 0, \quad t \geq t_0, \tag{1.7}$$

where $0 \leq p(t) < 1$.

To the best of our knowledge, the above oscillation results cannot be applied to study the case of $p(t) > 1$, and it seems to have few oscillation results for (1.1) when $p(t) > 1$.