

THE EULER EQUATION AND ONSAGER CONJECTURE^{*†}

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Abstract

In this paper, we introduce the progress of the Euler equation and Onsager conjecture. We also introduce the Euler's life, the researches about the incompressible Euler equation, and the Onsager conjecture.

Keywords incompressible Euler equation; Onsager conjecture; boundary layer; Lax pair; inviscid limit

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As the number theory, the Euler equation in the incompressible flow is the source of many branches of mathematics. Linear partial differential equation, dynamic system, nonlinear partial differential equation, geometric partial differential equation, harmonic analysis and the completely integrable system and other mathematical branches can be traced back to the Euler equation. Close to the weather forecast, as far as the supernova explosion theory, the influence of Euler equation has been run throughout all subjects outside mathematics. From the Euler equation, we can get the challenge problem and the inspiration.

1 Introduction about the Euler's Life

Leohard Euler was a Swiss mathematician, physicist, astronomer, logician and engineer. L. Euler was born on 15 April 1707, in Basel. He was died on 18 September 1783, in Saint Petersburg, Russian Empire. He was born in a clergyman's family.

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When he was 15, he got a bachelor's degree from the University of Basel, and got his master degree in the next year. Although his father wanted him to study theology, but he was interested in mathematics. He was guided by Bernoulli. At the age of 18, he gave up the idea of becoming a pastor, specialized in mathematics, and began to publish articles. In 1727, he was invited by Petersburg Academy to Russia. In 1731, he succeeded Bernoulli as a professor of Physics. He went into research with great energy. For 14 years in Russia, he did a great deal of work in analysis, theory of numbers, and mechanics. He also solved many practical problems such as geology and shipbuilding at the request of the Russian government. A lot of writing caused eye trouble that blinded him in the right eye in 1735. In 1741, he was invited to the Berlin Academy of Sciences by the great emperor of Prussia for 25 years. During his stay in Berlin, his researches were more extensive, involving planetary motion, rigid body transport, thermodynamics, demography, and mathematical models of fluids. These work and his mathematical research promoted each other. Euler's achievements in mathematical equations, surface geometry and other mathematical fields was pioneering at this time. In 1766 he returned to Petersburg. A serious illness make his left eye completely blind in 1771. However, because of his amazing memory and mental arithmetic skills, his creativity continued to work. He completed a great deal of scientific work through the discussion with his assistant until the last minute of his life. Euler is one of the most outstanding figures in Mathematics in eighteenth Century. Not only did he make great contributions to mathematics, but also he applied mathematics to almost the entire field of physics. He is an incomparable prolific author. He wrote a great deal of textbooks about mechanics, analysis, geometry, calculus. "The introduction to infinitesimal analysis", "The principles of calculus", and "The principles of integral calculus" all became the classical works of mathematics. In addition to textbooks, he wrote creative papers at the rate of eight hundred pages a year at his work. He wrote 856 papers and 31 monographs. His complete works are in 74 volumes.

Euler's greatest contribution was to expand the field of calculus and lay the foundation for the emergence and development of some important branches of analysis (such as infinite series, differential equations) and differential geometry.

Euler transformed an infinite series from a general tool into an important research project. His best result is to compute the number of functions in even numbers:

$$\xi(2k) = \sum_{n=1}^{\infty} n^{-2k} = a_{2k}\pi^{2k}.$$

He proved that a_{2k} was a rational number, and was expressed by Bernoulli numbers. He studied the harmonic series and calculated the Euler constant exactly