Ann. of Appl. Math.
33:4(2017), 364-378

LOCAL POLYNOMIAL DOUBLE-SMOOTHING ESTIMATION OF A CONDITIONAL DISTRIBUTION FUNCTION WITH DEPENDENT*†

Mimi Hong‡, Xianzhu Xiong
(*College of Math. and Computer Science, Fuzhou University, Fuzhou 350116, Fujian, PR China*)

Abstract

Based on the idea of local polynomial double-smoother, we propose an estimator of a conditional cumulative distribution function with dependent and left-truncated data. It is assumed that the observations form a stationary α-mixing sequence. Asymptotic normality of the estimator is established. The finite sample behavior of the estimator is investigated via simulations.

Keywords local polynomial double-smoother; conditional cumulative distribution function; left-truncated data; α-mixing; asymptotic normality

2000 Mathematics Subject Classification 62G05; 62N02

1 Introduction

Estimation of a conditional distribution function has many important applications. For example, estimation of a conditional quantile function can be obtained by inverting the estimation of the conditional distribution function. In addition, as [4] pointed out, conditional distribution estimation can be applied to construct prediction intervals for the next value in stationary time series.

The nonparametric estimation of a conditional distribution function has received much attention. For instance, in the independent and identically distributed (i.i.d.) case, [4] proposed two single-smoothing estimators (smoothing the covariates only) of the conditional distribution function, while [15,5,6] considered the local linear double-smoothing (smoothing the dependent variable and covariates) estimator of the conditional distribution function. [14] considered the local polynomial double-smoothing estimator of the conditional distribution function in time series data,

*This work was supported by National Natural Science Foundation of China (No.11301084), and Natural Science Foundation of Fujian Province (No.2014J01010).

†Manuscript received April 12, 2017; Revised June 26, 2017

‡Corresponding author. E-mail: 1114335900@qq.com

which extended the results of [15]. As [14] pointed out, compared with the single-smoothing estimator, the double-smoothing estimator not only appears closer to a distribution function, but also has more flexibility to reduce the mean-squared error when the optimal bandwidths are selected. In these papers, it is supposed that data is complete.

In some fields as reliability and survival analysis, the lifetime variables may not be completely observable, right-censored or left-truncated data are often encountered. In this paper we consider the case where the response variable is left-truncated. Left-truncated data often occurs in astronomy, epidemiology, biometry and economics, see [11,12]. For left-truncated data, [7] first proposed the kernel estimator (that is, the local constant double-smoothing estimator) of the conditional distribution function in the i.i.d. setting, and then obtained the kernel estimator of conditional quantile function by inversion. [7] also investigated asymptotic properties of two kernel estimators. [13] extended the results of [7] to dependent and left-truncated data. [8] proposed the local polynomial single-smoothing estimator of the conditional distribution function with dependent and left-truncated data and established the asymptotic normality of the estimator. Recently, in view of the advantage of local linear fitting and double-smoothing, [16] extended the local linear double-smoothing method of [15] to the left-truncated model. They proposed the local linear double-smoothing estimator of the conditional distribution function in the i.i.d. setting, and then obtained the local linear double-smoothing estimator of conditional quantile function by inversion. And they obtained the asymptotic normality of two local linear double-smoothing estimators.

Since the scenario with dependent data is an important one in lots of applications with survival data (see [1,13]), in this paper, we will consider the local polynomial double-smoothing (LPDS) estimator of the conditional distribution function in the dependent and left-truncated data, which extends the LPDS estimator of the conditional distribution function in [14] to the left-truncation model. Furthermore, the LPDS estimator here not only smooths the local polynomial single-smoothing estimator of [8], but also generalizes the local linear double-smoothing estimator of [16] in the i.i.d. setting to the LPDS estimator in the dependent data case. When left-truncated data is stationary and α-mixing, we establish the asymptotic normality of the LPDS estimator.

The rest of the paper is organized as follows. Section 2 recalls the random left-truncation model and introduces the LPDS estimator of the conditional distribution function. The asymptotic normality of the estimator is stated in Section 3, while the proof is given in Section 5. Finite-sample performance of the estimator is investigated by a simulation study in Section 4.