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A NEW CLASS OF SOLUTIONS OF VACUUM EINSTEIN'S FIELD EQUATIONS WITH COSMOLOGICAL CONSTANT*†

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Abstract

In this paper, a new class of solutions of the vacuum Einstein's field equations with cosmological constant is obtained. This class of solutions possesses the naked physical singularity. The norm of the Riemann curvature tensor of this class of solutions takes infinity at some points and the solutions do not have any event horizon around the singularity.

Keywords vacuum Einstein's field equations; cosmological constant; exact solutions; naked physical singularity

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1 Introduction

The cosmological constant problem is one of the most outstanding and unsolved problems in cosmology. It has been a focal point of interest [1-4]. From our point of view, the universe possesses a non-zero cosmological constant which is considered as the vacuum energy density. The cosmological term provides a repulsive force opposing the gravitational pull between the galaxies. The recent measurements of type Ia Supernovae observations [5,6] and findings from the anisotropy measurements of the cosmic microwave background by the Wilkinson Microwave Anisotropy Probe [7] have shown that our universe is undergoing an accelerated expansion. It is the most accepted idea that a mysterious dominant component dubbed dark energy leads to this cosmic acceleration. The cosmological constant is a strong candidate for dark energy which pushes the universe accelerated expansion.

Exact solutions of Einstein's field equations with cosmological constant, which offer an alternative and complementary approach to study cosmological models, have been investigated from time to time. As early as 1918, Kottler extended the

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Schwarzschild solution and obtained the static spherically symmetric exterior solution with cosmological constant. Kramer et al. got the Reissner-Nordsröm exterior solution with cosmological constant [8]. Xu, Wu and Huang extended the Florides' solution to the case with cosmological constant [9]. Recently, Nurowski provided the first examples of vacuum metrics with cosmological constant which have a twisting quadruple principal null direction [10]. Kamenshchik and Mingarelli found an exact solution of a Bianchi-I Universe in the presence of dust, stiff matter and a negative cosmological constant, generalising the well-known Heckmann-Schucking solution [11]. Landry, Abdelqader and Lake studied the McVittie solution with a negative cosmological constant and they found that cosmological constant $\Lambda < 0$ ensures collapse to a Big Crunch [12]. Zubairi and Weber [13] derived the modified Tolman-Oppenheimer-Volkoff equations which account for a finite value of the cosmological constant for spherically symmetric mass distributions.

In this paper, we consider the vacuum Einstein's field equations with cosmological constant of the following form

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu} = 0, \tag{1}$$

or equivalently

$$R_{\mu\nu} = \Lambda g_{\mu\nu}. \tag{2}$$

They are solved by presenting a special form of Lorentzian metric and taking a proper ansatz. It is shown that this new class of solutions possesses naked physical singularity. Moreover, the solutions presented in this paper extend the results of [14] to the case with cosmological constant.

2 The Solutions with Cosmological Constant

In the coordinate (t, x, y, z), consider the metric of the form

$$\mathrm{d}s^2 = A\mathrm{d}t^2 - \frac{1}{A}\mathrm{d}x^2 - B\mathrm{d}y^2 - y^2 B\mathrm{d}z^2, \tag{3}$$

where $A = A(t, x)$ and $B = B(t, x)$. It is easy to verify that the determinant of metric $(g_{\mu\nu})$ is given by

$$g \stackrel{\triangle}{=} \det(g_{\mu\nu}) = -B^2 y^2 < 0,$$

so the metric $(g_{\mu\nu})$ is Lorentzian.

In view of metric (3), the vacuum Einstein's field equations with cosmological constant (2) reduce to the following system