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DIAGNOSABILITY OF CAYLEY GRAPH NETWORKS GENERATED BY TRANSPOSITION TREES UNDER THE COMPARISON DIAGNOSIS MODEL*†

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Abstract

Diagnosability of a multiprocessor system is one important study topic. Cayley graph network $Cay(T_n, S_n)$ generated by transposition trees T_n is one of the attractive underlying topologies for the multiprocessor system. In this paper, it is proved that diagnosability of $Cay(T_n, S_n)$ is $n-1$ under the comparison diagnosis model for $n \geq 4$.

Keywords interconnection network; graph; diagnosability; comparison diagnosis model; Cayley graph

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1 Introduction

Many multiprocessor systems take interconnection networks (networks for short) as underlying topologies and a network is usually represented by a graph where vertices represent processors and edges represent communication links between processors. We use graphs and networks interchangeably. For a system, study on the topological properties of its network is important. Furthermore, some processors may fail in studying the system, so processor fault identification plays an important role for reliable computing. The first step to deal with faults is to identify the faulty processors from the testing of the fault-free ones. The identification process is called

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the diagnosis of the system. A system is said to be t-diagnosable if all faulty processors can be identified without replacement, provided that the number of presenting faults does not exceed t. The diagnosability of a system G is the maximum value of t such that G is t-diagnosable [4,5,7,11,19,20].

Several diagnosis models were proposed to identify the faulty processors. One major approach is the PMC diagnosis model introduced by Preparata et al. [13]. The diagnosis of the system was achieved through two linked processors by testing each other. Another important model, namely the comparison diagnosis model (MM model), was proposed by Maeng and Malek [12]. In the MM model, to diagnose a system, a vertex sends the same task to two of its neighbors, and then compares their responses. Cayley graph network $Cay(T_n, S_n)$ generated by transposition trees T_n is one of the attractive underlying topologies for the multiprocessor system. The star graph and the bubble-sort graph are two special cases of $Cay(T_n, S_n)$ [1]. In [18], Zheng et al. proved the n-dimensional star graph is $(n-1)$-diagnosable under the comparison diagnosis model when $n \geq 4$. In [8], Lee and Hsieh proved the n-dimensional bubble-sort is $(n-1)$-diagnosable under the comparison diagnosis model when $n \geq 4$. In this paper, the diagnosability of $Cay(T_n, S_n)$ under the comparison diagnosis model is studied. It is proved that $Cay(T_n, S_n)$ is $(n-1)$-diagnosable under the comparison diagnosis model when $n \geq 4$.

2 Preliminaries

2.1 The MM* model

In the MM model [12,17], to diagnose a system G, a vertex sends the same task to two of its neighbors, and then compares their responses. To be consistent with the MM model, we have the following assumptions:

a. All faults are permanent.

b. A faulty processor produces incorrect outputs for each of its given tasks.

c. The output of a comparison performed by a faulty processor is unreliable.

d. Two faulty processors given the same input and task do not produce the same output.

The comparison scheme of a system G is modeled as a multigraph, denoted by $M(V(G), L)$, where L is the labeled-edge set. A labeled edge $(u, v)_w \in L$ represents a comparison in which two vertices u and v are compared by a vertex w, which implies $uw, vw \in E(G)$. The collection of all comparison results in $M(V(G), L)$ is called the syndrome, denoted by σ^*, of the diagnosis. If the comparison $(u, v)_w$ disagrees, then $\sigma^*((u, v)_w) = 1$; otherwise, $\sigma^*((u, v)_w) = 0$. Hence, a syndrome is a function from L to $\{0, 1\}$. The MM* model, denoted by $M(V(G), L^*)$, is a special case of the