Ann. of Appl. Math.
32:3(2016), 234-248

SCALE-TYPE STABILITY FOR NEURAL NETWORKS WITH UNBOUNDED TIME-VARYING DELAYS*†

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Abstract

This paper studies scale-type stability for neural networks with unbounded time-varying delays and Lipschitz continuous activation functions. Several sufficient conditions for the global exponential stability and global asymptotic stability of such neural networks on time scales are derived. The new results can extend the existing relevant stability results in the previous literatures to cover some general neural networks.

Keywords global asymptotic stability; global exponential stability; neural networks; on time scales

2000 Mathematics Subject Classification 92B20

1 Introduction

Consider a general class of neural networks with unbounded time-varying delays on time scales:

$$x_i^{\Delta}(t) = -c_i x_i(t) + \sum_{j=1}^{n} a_{ij} f_j(x_j(t)) + \sum_{j=1}^{n} b_{ij} g_j\big(x_j(t-\tau_{ij}(t))\big), \tag{1.1}$$

where $x_i(t)$ corresponds to the state of the ith unit at time $t \in \mathbb{T}$, $f_j(x_j)$ and $g_j(x_j)$ are the activation functions of the jth unit, $c_i > 0$ represents the rate with which the ith unit will reset its potential to the resting state in isolation when disconnected from the network, $\tau_{ij}(t)$ corresponds to the transmission delay which satisfies $\tau_{ij}(t) \geq 0$, a_{ij} and b_{ij} denote the strength of the jth neuron on ith unit at time t and $t - \tau_{ij}(t)$, $i, j \in N$, where $N = \{1, 2, \cdots, n\}$. In this paper, we make some basic assumptions:

*This research was supported by National Natural Science Foundation of China under Grant 61573005 and 11361010, the Foundation for Young Professors of Jimei University and the Foundation of Fujian Higher Education (JA11154,JA11144).

†Manuscript received April 21, 2016; Revised June 7, 2016

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1) $f_i(0) = g_i(0) = 0$;

2) There exist constants $l_i > 0,\ k_i > 0$ such that for any $r_1, r_2, r_3, r_4 \in R$

$$|f_i(r_1) - f_i(r_2)| \le l_i |r_1 - r_2|,$$
$$|g_i(r_3) - g_i(r_4)| \le k_i |r_3 - r_4|;$$

3) $\hat{\tau}_{ij} : \mathbb{T} \to \mathbb{T}$ with $\hat{\tau}_{ij} = t - \tau_{ij}(t)$.

Denote $\|x\|_q$ as the vector q-norm of the vector x with q satisfies $1 \le q < \infty$. $\|x\|_\infty = \max\limits_{i\in N} |x_i|$ is the vector infinity norm. Denote $\|A\|_q$ as the q-norm of the matrix A induced by the vector q-norm. For simplification, we denote $(-\infty, t_0]_{\mathbb{T}}$ by $(-\infty, t_0] \cap \mathbb{T}$.

For any $t_0 \ge 0$, the initial condition of the neural network model (1.1) is assumed to be

$$x_i(s) = \Phi_i(s), \quad s \in (-\infty, t_0]_{\mathbb{T}} \ \text{ and } \ i \in N. \tag{1.2}$$

In stability analysis of neural networks, the qualitative properties primarily concerned are the uniqueness, global stability, robust stability, and absolute stability of their equilibria. In [6] and [8], global asymptotic and exponential stability were given for neural networks without time delays. The case of constant time delay was also studied in [2,7]. In [3,9], the authors discussed the case of bounded time-varying delay. In addition, the authors in [4] described the case of unbounded time-varying delay, that gave several sufficient conditions for the global exponential stability. In [10], several algebraic criterions for stability were obtained by constructing proper Lyapunov functions and employing Young inequality.

Recently, people have paid attention to the neural network models on time scales, and some of them have got some important results, such as [11-23]. In [12], by using the contraction mapping theorem and Gronwall's inequality on time scales, the authors established some sufficient conditions on the existence and exponential stability of periodic solutions of a class of stochastic neural networks on time scales. In [14,16,18], the authors paid attention to the periodic solutions of a class of neural networks delays on time scales. Based on contraction principle and Gronwall-Bellmans inequality, some new results for the existence and exponential stability of almost periodic solution of a general type of delay neural networks with impulsive effects were established in [15]. The problem on the global exponential stability of neural networks on time scales was considered in [13,22,23]. In [17,19-21], global exponential stability of networks with time-varying delays on time scales were considered.

In this paper, we consider a general neural network model on time scales. By using different methods, several sufficient conditions for the global asymptotic sta-