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OPTIMAL DECAY RATE OF THE COMPRESSIBLE QUANTUM NAVIER-STOKES EQUATIONS*†

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Abstract

For quantum fluids governed by the compressible quantum Navier-Stokes equations in $\mathbb{R}^3$ with viscosity and heat conduction, we prove the optimal $L^p - L^q$ decay rates for the classical solutions near constant states. The proof is based on the detailed linearized decay estimates by Fourier analysis of the operators, which is drastically different from the case when quantum effects are absent.

Keywords compressible quantum Navier-Stokes equations; optimal decay rates; energy estimates

2000 Mathematics Subject Classification 35M20; 35Q35

1 Introduction

Let us consider the following classical hydrodynamic equations in $\mathbb{R}^3$ describing the motion of the electrons in plasmas by omitting the electric potential

$$\begin{cases} \dfrac{\partial n}{\partial t} + \mathrm{div}\Pi = 0, & (1.1\text{a}) \\ \dfrac{\partial \Pi}{\partial t} + \mathrm{div}(nu \otimes u - P) = 0, & (1.1\text{b}) \\ \dfrac{\partial W}{\partial t} + \mathrm{div}(uW - Pu + q) = 0, & (1.1\text{c}) \end{cases}$$

where n is the density, $u = (u_1, u_2, u_3)$ is the velocity, $\Pi = (\Pi_1, \Pi_2, \Pi_3)$ and $\Pi_j = nu_j$

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is the momentum density, $P=(P_{ij})_{3\times 3}$ is the stress tensor, W is the energy density and $q=-\kappa\nabla T$ is the heat flux and T is the temperature. This system also emerges from descriptions of the motion of the electrons in semiconductor devices, with the electrical potential and the relaxation omitted [4].

In this paper, we consider the following case. The stress tensor is given by

$$P=-nT\mathbb{I}+\frac{\hbar^2 n}{12}\nabla^2\ln n+\mathbb{S},$$

where $\mathbb{I}$ is the identity matrix and $\mathbb{S}$ is the viscous part of the stress tensor given by

$$\mathbb{S}=\mu(\nabla u+(\nabla u)^{\mathrm{T}})+\delta(\mathrm{div}u)\mathbb{I},$$

where $\mu>0$ and δ are the primary coefficients of viscosity and the second coefficients of viscosity, respectively, satisfying $2\mu+3\delta\geq 0$. The energy density W is given by

$$W=\frac{3}{2}nT+\frac{1}{2}nu^2-\frac{\hbar^2 n}{24}\Delta\ln n.$$

The quantum correction to the stress tensor was proposed by Ancona and Tiersten [2] on general thermodynamical grounds and derived by Ancona and Iafrate [1] in the Wigner formalism. The quantum correction to the energy density was first derived by Wigner [14]. See also [5]. With these quantum corrections ($\hbar>0$), system (2.7) is called the compressible quantum Navier-Stokes (CQNS) equations. When $\hbar=0$, it reduces to the standard compressible Navier-Stokes (CNS) equations and was studied by Matsumura and Nishida [9] for the existence of smooth small solutions.

Obviously, $(n,u,T)=(1,0,1)$ is a solution for (2.7). To show the existence of small solutions near $(1,0,1)$, we consider $(\rho,u,\theta)=(n-1,u,T-1)$ and transform (2.7) into the following quantum hydrodynamic equation

$$\begin{cases}\partial_t\rho+u\cdot\nabla\rho+(1+\rho)\mathrm{div}u=0, & \text{(1.2a)}\\ \partial_t u-\dfrac{\mu}{\rho+1}\Delta u-\dfrac{\mu+\delta}{\rho+1}\nabla\mathrm{div}u=-u\cdot\nabla u-\nabla\theta-\dfrac{\theta+1}{\rho+1}\nabla\rho+\dfrac{\hbar^2}{12}\dfrac{\Delta\nabla\rho}{\rho+1} & \\ \qquad\qquad -\dfrac{\hbar^2}{3}\dfrac{\mathrm{div}(\nabla\sqrt{\rho+1}\otimes\nabla\sqrt{\rho+1})}{\rho+1}, & \text{(1.2b)}\\ \partial_t\theta-\dfrac{2\kappa}{3(1+\rho)}\Delta\theta=-u\cdot\nabla\theta-\dfrac{2}{3}(\theta+1)\nabla\cdot u+\dfrac{\hbar^2}{36(1+\rho)}\mathrm{div}((1+\rho)\Delta u) & \\ \qquad\qquad +\dfrac{2}{3(1+\rho)}\left\{\dfrac{\mu}{2}|\nabla u+(\nabla u)^T|^2+\delta(\mathrm{div}u)^2\right\}. & \text{(1.2c)}\end{cases}$$

Recently, we obtained the following global existence result of small smooth solutions in [12].