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THE SYMMETRIC POSITIVE SOLUTIONS OF $2n$-ORDER BOUNDARY VALUE PROBLEMS ON TIME SCALES*†

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Abstract

In this paper, we are concerned with the symmetric positive solutions of a $2n$-order boundary value problems on time scales. By using induction principle, the symmetric form of the Green's function is established. In order to construct a necessary and sufficient condition for the existence result, the method of iterative technique will be used. As an application, an example is given to illustrate our main result.

Keywords symmetric positive solutions; boundary value problems; induction principle; time scales; iterative technique

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1 Introduction

The theory of measure chains (time scales) was first introduced by Stefan Hilger in his Ph.D. thesis (see [1]) in 1988. Although it is a new research area of mathematics, it has already caused a lot of applications, e.g., insect population models, neural networks, heat transfer and epidemic models (see [2,3]). Some of these models can be found in [4-6]. Such as in [5], Q.K. Song and Z.J. Zhao discussed the problem on the global exponential stability of complex-valued neural networks with both leakage delay and time-varying delays on time scales. By constructing appropriate Lyapunov-Krasovskii functionals and using matrix inequality technique, a delay-dependent condition assuring the global exponential stability for the considered neural networks was established.

In the past few years, more and more scholars concentrated on a positive solution of boundary value problems for differential equations on time scales (see [7-12]). In

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[13,14], by using some fixed point theorems, the existences of pseudo-symmetric solutions of dynamic equations on time scales were obtained. In [15,16], the fourth order integral boundary value problems on time scales for an increasing homeomorphism and homomorphism were discussed. Recently, the conditions for the existence of symmetric positive solutions of boundary value problems were constructed in [17,18]. By applying an iterative technique, the existence and uniqueness of symmetric positive solutions of the $2n$-order nonlinear singular boundary value problems of differential equation

$$\begin{cases}(-1)^n u^{(2n)}(t) = f(t, u(t)), \quad t \in (0,1),\\ u^{(2k)}(0) = u^{(2k)}(1) = 0, \quad k = 0, 1, 2, \cdots, n-1,\end{cases}$$

were obtained.

In this paper, we are concerned with the existence of symmetric positive solutions of the following $2n$-order boundary value problems (BVP) on time scales

$$\begin{cases}(-1)^n u^{\Delta^{2n}}(t) = f(\sigma(t), u^{\sigma}(t)), \quad t \in [0, \sigma(1)],\\ u^{\Delta^{2i}}(0) = u^{\Delta^{2i}}(\sigma(1)) = 0, \quad 0 \le i \le n-1,\end{cases} \tag{1}$$

where $f : [0, \sigma(1)] \times [0, \infty) \to [0, \infty)$ is continuous and $f(t, u)$ may be singular at $u = 0$, $t = 0$ (and/or $t = \sigma(1)$). If a function $u : [0, \sigma(1)] \to \mathbb{R}$ is continuous and satisfies $u(t) = u(\sigma(1) - t)$ for $t \in [0, \sigma(1)]$, then we say that $u(t)$ is symmetric on $[0, \sigma(1)]$. By a symmetric positive solution of BVP (1), we mean a symmetric function $u \in C^{2n}[0, \sigma(1)]$ such that $(-1)^i u^{\Delta^{2i}}(t) > 0$ for $t \in (0, \sigma(1))$ and $i = 0, 1, \cdots, n-1$, and $u(t)$ satisfies BVP (1). We assume that $\sigma(1)$ and 0 are all right dense. Throughout this paper we let $\mathbb{T}$ be any time scale (nonempty closed subset of $\mathbb{R}$) and $[a, b]$ be a subset of $\mathbb{T}$ such that $[a, b] = \{t \in \mathbb{T} : a \le t \le b\}$. $\mathbb{T}$ satisfies

$$\sigma(a - \sigma(b)) = \sigma(a) - \sigma(b), \tag{2}$$

and it is easy to see that $\mathbb{T} = \mathbb{R}$ or $\mathbb{T} = h\mathbb{Z}$ satisfies (2). And thus $\widetilde{\mathbb{T}} = \{\sigma(t) | t \in \mathbb{T}\} = \mathbb{T}$.

2 Preliminary

Before discussing the problems of this paper, we introduce some basic materials for time scales which are useful in proving our main results. These preliminaries can be found in [17-20].

Lemma 2.1[20] (Substitution) *Assume that $\nu : \mathbb{T} \to \mathbb{R}$ is strictly increasing and $\widetilde{\mathbb{T}} := \nu(\mathbb{T})$ is a time scale. If $f : \mathbb{T} \to \mathbb{R}$ is an rd-continuous function and ν is differentiable with rd-continuous derivative, then for $a, b \in \mathbb{T}$,*