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Traveling Wave Solutions in an Integrodifference Equation with Weak Compactness*

Shuxia Pan[1] and Guo Lin[2,†]

Abstract This article studies the existence of traveling wave solutions in an integrodifference equation with weak compactness. Because of the special kernel function that may depend on the Dirac function, traveling wave maps have lower regularity such that it is difficult to directly look for a traveling wave solution in the uniformly continuous and bounded functional space. In this paper, by introducing a proper set of potential wave profiles, we can obtain the existence and precise asymptotic behavior of nontrivial traveling wave solutions, during which we do not require the monotonicity of this model.

Keywords Generalized upper and lower solutions, Traveling wave map, Minimal wave speed, Decay behavior.

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1. Introduction

When some species with non-overlapping generations are concerned, their spatial dispersal and growth often occur at different stages of the species, and many plants have the feature. In population dynamics, Mollison [14] and Weinberger [16] proposed some discrete time models equipping with spatial variables to describe these phenomena, which are integrodifference equations [11]. One typical integrodifference equation takes the following iterative form

$$w_{n+1}(x) = \int_{\mathbb{R}} b(w_n(y))k(x-y)dy,\ x \in \mathbb{R},\ n = 0, 1, 2, \cdots. \tag{1.1}$$

Regarding (1.1) as a model in population dynamics, then $w_n(x)$ often denotes the density of the species at location x of the nth generation, b is the birth function while k reflects the spatial movement law and may be a probability distribution that is also the so-called kernel function. Since Weinberger [16], the traveling wave solutions of (1.1) have been widely studied, see some results by Bourgeois et al. [1], Fang and Zhao [2], Hsu and Zhao [3], Kot [5], Li et al. [7], Liang and Zhao [9], Wang and Castillo-Chavez [15], Weinberger et al. [17], Yi et al. [18]. Here, a traveling wave

†the corresponding author.
Email address: shxpan@yeah.net (S. Pan), ling@lzu.edu.cn (G. Lin)
[1]School of Science, Lanzhou University of Technology, Lanzhou, Gansu 730050, China
[2]School of Mathematics and Statistics, Lanzhou University, Lanzhou, Gansu 730000, China
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solution of (1.1) is a special solution taking the form

$$w_n(x) = \varphi(t),\ t = x + cn \in \mathbb{R},$$

in which φ is the wave profile that propagates in the spatial media at the constant speed c, and $t = x + cn$ is the traveling wave coordinate. That is, a traveling wave solution must satisfy

$$\varphi(t) = \int_{\mathbb{R}} b(\varphi(y))k(t-c-y)dy := B(\varphi)(t),\ t \in \mathbb{R}.$$

Clearly, a traveling wave solution is a fixed point of B in proper functional space.

When b is continuous and k is Lebesgue measurable and integrable, then we see that the traveling wave map $B(\varphi)(t)$ is equicontinuous if $\varphi(t)$ belongs to proper bounded continuous functional set. Based on such a property of the map B, it is possible to obtain necessary smoothness in continuous functional space, which implies that fixed point theorem can be applied to study the existence of traveling wave solutions [3,10]. With the help of fixed point theorem, the existence of traveling wave solutions may be obtained by the existence of proper generalized upper and lower solutions [3, 10]. Of course, some other methods were also utilized to study the existence of nonconstant traveling wave solutions in [7, 9, 15, 17, 18], in which proper smoothness or monotone conditions of traveling wave maps are necessary.

However, in many examples, the kernel function is not Lebesgue measurable and integrable such that the smoothness of $B(\varphi)(t)$ encounters difficulty. In particular, considering a nondispersing (sessile) component, Lutscher [11, Section 12.4] proposed and studied some mathematical models, in which the kernel function may depend on the Dirac function. One example in [11, Section 12.4] and Li [6] takes the following form

$$u_{n+1}(x) = ru_n(x) + \int_{\mathbb{R}} f(u_n(y))k(x-y)dy,\ x \in \mathbb{R},\ n = 0, 1, 2, \cdots, \tag{1.2}$$

in which $r > 0$ is a constant formulating the dormant behavior of seeds, f is a function reflecting the newborn viable seeds, k is a probability distribution denoting the dispersal of seeds. Consider the traveling wave map

$$F_1(\varphi)(t) = r\varphi(t-c) + \int_{\mathbb{R}} f(\varphi(y))k(t-c-y)dy,$$

then we see that $F_1(\varphi)(t)$ may be not equicontinuous if $\varphi(t)$ belongs to proper bounded continuous functional set. Due to the deficiency of higher regularity of F_1, we can not directly utilize fixed point theorem as that in [3, 10].

With the help of propagation theory of monotone semiflows [9, 17], Pan et al. [13] obtained the minimal wave speed of traveling wave solutions in (1.2) if f is monotone. Recently, Pan [12] has studied the existence and the asymptotic behavior of traveling wave solutions of (1.2) by constructing proper wave profile set. More precisely, the author first introduced a set Y by a pair of upper and lower solutions of wave equation of (1.2), in which upper and lower solutions are given by the conclusion in [13]. Based on such a set Y, they studied the set

$$\overline{Y} = \bigcap_{n>0} \overline{Co(F_1^n[Y])},$$