Journal of Nonlinear Modeling and Analysis
Volume 3, Number 4, December 2021, 617–646
http://jnma.ca; http://jnma.ijournal.cn
DOI:10.12150/jnma.2021.617

# Nonlocal Interaction Induces the Self-organized Mussel Beds*

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**Abstract** Mussel beds are important habitats and food sources for biodiversity in coastal ecosystems. The predation of mussel on algae depends not only on the current time and location, but also on the quantity of algae at other spatial location and time. To know the impacts of such predation behavior on the dynamics of mussel beds well, we pose a reaction-diffusion mussel-algae model coupling nonlocal interaction with kernel function. By calculating the critical conditions of Hopf bifurcation and Turing bifurcation, the conditions for the generation of Turing pattern are obtained. We find that the diffusion rate and predation rate of mussels have effect on the structure and density of spatial pattern of mussels under the nonlocal interaction, and the predation rate of mussels can produce different pattern types, while the diffusion rate plays a more important role on the pattern density. Moreover, the nonlocal interaction promotes the stability of the mussel beds. These results suggest that the nonlocal interaction between mussels and algaes is one of the important mechanisms for the formation of the spatial structure of mussel beds.

**Keywords** Nonlocal interaction, Mussel-algae system, Hopf bifurcation, Turing pattern, Multi-scale analysis.

**MSC(2010)** 35B32, 35B36, 37G10, 92B05.

## 1. Introduction

Mussel beds provide an important habitat of biodiversity and food source of marine ecosystem, and the stability of mussel beds play an important role in marine

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*The authors were supported by the National Key Research and Development Program of China (Grant No. 2018YFE0109600), National Natural Science Foundation of China (Grant Nos. 42075029 and 11671241), Program for the Outstanding Innovative Teams (OIT) of Higher Learning Institutions of Shanxi, Natural Science Foundation of Shanxi Province (Grant No. 201801D221003), China Postdoctoral Science Foundation (Grant Nos. 2017M621110 and 2019T120199) and Outstanding Young Talents Support Plan of Shanxi Province.

ecosystem. Therefore, it is of great significance to mussel beds [20, 22, 23, 27, 45]. Mussels that accumulating in soft sediments usually survive by feeding on algaes which in the lower water layers [38, 42]. Since mussel is largely dependent on the density of the algae for their survival, and the low density of algae maybe leads to the depletion of the mussel beds [8]. In addition, in order to compete for the algae, the mussel usually tend to cluster in the water where the algae gathers [23]. Given that the algae is the main food source of mussel population, many ecologists and mathematicians have thus focused on the interaction between mussel and algae [3, 23, 30, 40].

Experimental studies have shown that mussel gathers to form different spatial distributions due to competition and survival [30], and such phenomenon is called a self-organizing behavior, which is of great significance to the stability existence and restoration of ecological environment [12,18,21,32–34,44]. In fact, mussel population develops self-organizing patterns in two different spatial scales: mussel population forms linear clusters driven by individuals' aggregation behaviors in small spatial scale, which is a rapid process, namely about a day [22,25]; while mussel population produce large, regular zones perpendicular to the flow of water in large spatial scale based on the ecological feedback mechanism (local promotion of algae and large-scale competition of mussel) [30]. The coupling of two forms of self-organization enhances the persistence and robustness of mussel beds compared to non-organized river beds. Therefore, it is necessary to study pattern formation mechanism of mussel beds [30]. Van de Koppel et al. studied the mussel-algae system by using the reaction-diffusion-advection equation [23] for the first time:

$$\begin{cases} \dfrac{\partial A}{\partial t} = (A_{up} - A)f - \dfrac{c}{h}AM - V\dfrac{\partial A}{\partial X}, \\ \dfrac{\partial M}{\partial t} = ecAM - d_M\dfrac{k_M}{k_M + M}M + D\dfrac{\partial^2 M}{\partial X^2}. \end{cases} \tag{1.1}$$

This paper mainly reported regular spatial pattern of young mussel beds on soft sediments in the Wadden Sea [24]. It has been also proposed that the scale dependent mechanism leads to the spatial self-organization of mussels, which provides a possible explanation for the spatial pattern. Based on this model, Wang et al. analyzed the differential-flow instability conditions that cause the formation of spatial patterns and the influence of parameters on the spatial pattern, and it has also been found that the spatial pattern is the result of the interaction of nonlinear terms [40]. The following literature mainly considered that movement of the algae depends on the flow of water, while ignore the random diffusion of the algae. Thus, a new model with respect to random diffusion of the algae was proposed by Cangelosi et al. [3]:

$$\begin{cases} \dfrac{\partial A}{\partial t} = (A_{up} - A)\rho - \dfrac{c}{H}AM - V\dfrac{\partial A}{\partial X} + D_A\triangle A, \\ \dfrac{\partial M}{\partial t} = ecAM - d_M\dfrac{k_M}{k_M + M}M + D_M\triangle M, \end{cases} \tag{1.2}$$

the interaction between the young mussel beds and algaes has been studied by using the weak nonlinear diffusion instability analysis method, and the transverse diffusion coefficient of algae which introduced in this system has been obtained by using the spectral analysis method [3].

In fact, many scholars have studied the spatial pattern formed by biological interaction, which has important significance for exploring the possibility of coexis-