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The Number of Limit Cycles in a Class of Piecewise Polynomial Systems

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Abstract In this paper, we pay attention to the number of limit cycles for a class of piecewise smooth near-Hamiltonian systems. By using the expression of the first order Melnikov function and some known results about Chebyshev systems, we study upper bound of the number of limit cycles in Hopf bifurcation and Poincaré bifurcation respectively.

Keywords Piecewise smooth system, Melnikov function, ECT-system, Limit cycle.

MSC(2010) 37E05, 34C07.

1. Introduction

As we know, the second part of Hilbert's 16th problem [5] is to estimate the number of limit cycles in a planar system

$$\begin{cases} \dot{x} = P_n(x, y), \\ \dot{y} = Q_n(x, y), \end{cases}$$

where $P_n(x, y)$ and $Q_n(x, y)$ represent nth-degree polynomials in (x, y) and to investigate their distributions. In 1977, Arnold [1] first proposed the weakened Hilbert's 16th problem, which is to study the maximum number of zeros of the first order Melnikov function in the following near-Hamiltonian system

$$\begin{cases} \dot{x} = H_y(x, y) + \varepsilon f(x, y), \\ \dot{y} = -H_x(x, y) + \varepsilon g(x, y), \end{cases}$$

where H, f and g are polynomials in (x, y), and $\varepsilon > 0$ is small. This problem is still open. Many mathematicians have done a lot of researches on limit cycle bifurcations.

In recent years, stimulated by non-smooth phenomena in the real world, piecewise smooth systems have been widely investigated (see [2, 3, 6, 7, 9, 12, 13] for example). The authors [9] considered the piecewise near-Hamiltonian system on the

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plane

$$
\dot{x} = \begin{cases} H_y^+(x,y) + \varepsilon p^+(x,y,\delta), & x \geq 0, \\ H_y^-(x,y) + \varepsilon p^-(x,y,\delta), & x < 0, \end{cases}
\\
\dot{y} = \begin{cases} -H_x^+(x,y) + \varepsilon q^+(x,y,\delta), & x \geq 0, \\ -H_x^-(x,y) + \varepsilon q^-(x,y,\delta), & x < 0, \end{cases} \tag{1.1}
$$

where $H^{\pm}, p^{\pm}$ and $q^{\pm}$ are C^{∞}, $\varepsilon > 0$ is small, and $\delta \in D \subset \mathbb{R}^m$ is a vector parameter with D a compact set. Suppose that $(1.1)|_{\varepsilon=0}$ has a family of periodic orbits around the origin and satisfies the following two assumptions [3, 9].

Assumption (I). There exist an interval $J = (\alpha, \beta)$, and two points $A(h) = (0, a(h))$ and $A_1(h) = (0, a_1(h))$ such that for $h \in J$,

$$
H^+(A(h)) = H^+(A_1(h)) = h, \; H^-(A(h)) = H^-(A_1(h)),
$$

$$
H_y^{\pm}(A(h)) H_y^{\pm}(A_1(h)) \neq 0, \; a(h) > a_1(h).
$$

Assumption (II). The equation $H^+(x,y) = h, \; x \geq 0$, defines an orbital arc L_h^+ starting from $A(h)$ and ending at $A_1(h)$; the equation $H^-(x,y) = H^-(A_1(h)), \; x \leq 0$, defines an orbital arc L_h^- starting from $A_1(h)$ and ending at $A(h)$, such that $(1.1)|_{\varepsilon=0}$ has a family clockwise periodic orbits $L_h = L_h^- \bigcup L_h^+, \; h \in J$.

The first order Melnikov function was defined and its formula was given in [9]. In fact, by Theorem 1.1 in [9] and Lemma 2.2 in [7], the first order Melnikov function of system (1.1) has the form

$$
M(h,\delta) = M^+(h,\delta) + \frac{H_y^+(A)}{H_y^-(A)} M^-(h,\delta), \tag{1.2}
$$

where

$$
M^{\pm}(h,\delta) = \int_{L_h^{\pm}} q^{\pm} dx - p^{\pm} dy.
$$

In [13], the authors studied a piecewise smooth near-Hamiltonian system of the form

$$
\begin{cases} \dot{x} = y + \varepsilon p(x,y,\delta), \\ \dot{y} = -g(x) + \varepsilon q(x,y,\delta), \end{cases} \tag{1.3}
$$

where

$$
g(x) = \begin{cases} a_1 x + a_0, \; x \geq 0, \\ b_1 x + b_0, \; x < 0, \end{cases}
$$

$$
p(x,y,\delta) = \begin{cases} p^+(x,y,\delta) = \sum_{i+j=0}^{n} a_{ij}^+ x^i y^j, \; x \geq 0, \\ p^-(x,y,\delta) = \sum_{i+j=0}^{n} a_{ij}^- x^i y^j, \; x < 0, \end{cases}
$$

$$
q(x,y,\delta) = \begin{cases} q^+(x,y,\delta) = \sum_{i+j=0}^{n} b_{ij}^+ x^i y^j, \; x \geq 0, \\ q^-(x,y,\delta) = \sum_{i+j=0}^{n} b_{ij}^- x^i y^j, \; x < 0, \end{cases}
$$