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Spatiotemporal Dynamic Analysis in a Time-space Discrete Brusselator Model

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Abstract In this paper, we study the spatiotemporal patterns of a Brusselator model with discrete time-space by using the coupled mapping lattice (CML) model. The existence and stability conditions of the equilibrium point are obtained by using linear stability analysis. Then, applying the center manifold reduction theorem and the bifurcation theory, the parametric conditions of the flip and the Neimark-Sacker bifurcation are described respectively. Under space diffusion, the model admits the Turing instability at stable homogeneous solutions under some certain conditions. Two nonlinear mechanisms, including flip-Turing instability and Neimark-Sacker-Turing instability, are presented. Through numerical simulation, periodic windows, invariant circles, chaotic phenomenon and some interesting spatial patterns are found.

Keywords Discrete Brusselator model, Bifurcation, Turing instability, Couple map lattice.

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1. Introduction

In 1952, Turing [29] proposed the reaction-diffusion coupling equation for the first time, and obtained that the system transforms from a stable process to an unstable process under the action of diffusion. He believed that the diffusion may destroy the spatial homogeneous states and lead to non-homogeneous spatial patterns. This instability is known as the Turing instability, which has been also known as the diffusion-driven instability. In recent decades, Turing instability has been applied to many fields such as biology, physics, chemistry, etc. In chemical systems, Turing patterns can be produced by a number of reactions such as the famous Brusselator model.

Brusselator model, as an autocatalytic reaction, was proposed by Prigogine and Lefever [23] in the 1960s. Since then, it has attracted the attention of many scholars, and detailed theoretical analysis and experimental research have been carried out on the dynamic behavior of Brusselator system in continuous time and space. It has been found that Hopf bifurcation and Turing instability would occur in the continuous Brusselator system. The combination of Hopf bifurcation and Turing

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instability can produce some patterns. For example, it refers to [7, 9, 11, 15, 16, 18] and their relevant literature.

In many cases, however, it is not continuous in time. For example, in [25], the nonlinear dynamical behaviors of two discrete-time versions of the continuous time Brusselator model were considered. Moreover, in [6], the dynamics of Brusselator model with discrete time were studied, and a new chaos control method was proposed based on bifurcation theory and center manifold theorem to control the chaos of Brusselator model with discrete time under the influence of flip and Hopf bifurcation.

As we know, there are many methods that can establish the discrete model. Since the coupled mapping lattice (CML) model can discretize the corresponding continuous reaction diffusion model, it has been widely used (see [3,13,28,31,33–35]). The continuous system is discretized by CML model, which leads to the unique nonlinear mechanism and characteristics of the time-space discrete Brusselator system. The most important nonlinear mechanisms are the various bifurcation behaviors, including flip bifurcation and Neimark-Sacker bifurcation. As a unique bifurcation phenomenon of discrete Brusselator system, flip bifurcation can lead to the formation of the path to chaos accompanied by period-doubling process. The combination of flip bifurcation and Turing instability can lead to the formation of complex pattern patterns. As described in [17], Neimark-Sacker bifurcation in the discrete system is the discrete counterpart of the Hopf bifurcation that occurs in the continuous system. The combination of Neimark-Sacker bifurcation and Turing instability gives rise to Neimark-Sacker-Turing instability, resulting in periodic orbits, invariant circles, chaotic attractors and other complex patterns, which are of exploratory significance. Therefore, we will consider the dynamical behavior of the time-space discrete Brusselator model in this paper.

The center manifold reduction and normal form theory are frequently used in the study of the bifurcation. For example, in [12], the stability and local Hopf bifurcation of Leslie-Gower predator-prey system with discrete distributed delay were studied. By using the center manifold reduction and normal form theory, formulas were obtained to determine the stability and direction of periodic solutions of Hopf bifurcation. Similarly, the classical Lotka-Volterra predator-prey model was studied in [8]. The results showed that the existence of time delay will change the stability of the equilibrium point, while the fear effect will stabilize the equilibrium point. Using the the center manifold reduction and normal form theory, formula for determining the stability and direction of Hopf bifurcation periodic solution was derived.

The paper is organized as follows. In Section 2, the time-space discrete Brusselator model is developed, and the existence and stability conditions of the equilibrium points are obtained. In Section 3, we give the parametric conditions for the Neimark-Sacker, flip and Turing bifurcation to occur. In Section 4, numerical simulations are presented to illustrate the theoretical results. The Turing instability region is also identified. In Section 5, we draw some conclusions.