Numer. Math. Theor. Meth. Appl. Vol. **15**, No. 3, pp. 592-619
doi: 10.4208/nmtma.OA-2021-0131 August 2022

Two-Step Two-Sweep Modulus-Based Matrix Splitting Iteration Method for Linear Complementarity Problems

Maryam Bashirizadeh and Masoud Hajarian*

Department Applied Mathematics, Faculty of Mathematical Sciences, Shahid Beheshti University, Tehran, Iran

Received 22 July 2021; Accepted (in revised version) 20 February 2022

Abstract. Linear complementarity problems have drawn considerable attention in recent years due to their wide applications. In this article, we introduce the two-step two-sweep modulus-based matrix splitting (TSTM) iteration method and two-sweep modulus-based matrix splitting type II (TM II) iteration method which are a combination of the two-step modulus-based method and the two-sweep modulus-based method, as two more effective ways to solve the linear complementarity problems. The convergence behavior of these methods is discussed when the system matrix is either a positive-definite or an H_+-matrix. Finally, numerical experiments are given to show the efficiency of our proposed methods.

AMS subject classifications: 65F10, 65F15

Key words: Linear complementarity problem, modulus-based method, two-step, two sweep, H_+-matrix, convergence.

1. Introduction

Linear complementarity problems attract many researchers' attention as a current research field. These problems arise typically in various practical applications from economics, engineering, and sciences. The study of numerical techniques to solve these problems is significant accordingly.

For the given pair of vector $q \in R^n$ and matrix $A \in R^{n\times n}$, linear complementarity problem $(\mathrm{LCP}(q,A))$ is to find a vector $z \in R^n$ such that

$$z \geq 0, \quad \omega := Az + q \geq 0, \quad z^T\omega = 0. \tag{1.1}$$

$\mathrm{LCP}(q,A)$ commonly involves optimization strategies such as linear or quadratic programming, optimal capital invariant stock, optimal stopping problems, and many more [7–12, 17, 23, 25, 29, 32, 43, 44, 48].

*Corresponding author. *Email addresses:* m_bashirizadeh@sbu.ac.ir (M. Bashirizadeh), m_hajarian@sbu.ac.ir (M. Hajarian)

Direct and iterative methods are two major categories of numerical methods which widely use for solving $\mathrm{LCPs}(q,A)$ [1, 3, 6, 13, 14, 21, 22, 28, 30, 38, 46]. Among them, modulus-based matrix splitting iteration methods have a special place [31, 37]. In 1981, Van Bokhoven [38] presented a modulus iteration method by reformulating the $\mathrm{LCP}(q,A)$ as an implicit fixed point equation for the first time. Afterward, based on the modulus iteration method [38] and the matrix splitting of the system matrix A, Bai [2] developed $\mathrm{LCP}(q,A)$ that led to a series of other modulus-based matrix splitting iteration methods by presenting a general framework of modulus-based matrix splitting iteration methods. In this regard, various modulus-based matrix splitting iteration methods have been proposed [4, 15, 16, 27, 43, 45, 50, 52, 53, 56, 57]. Convergence analysis of these methods for solving $\mathrm{LCP}(q,A)$ was discussed when the system matrix is either a positive-definite or an H_+-matrix.

In 2011, Zhang [49] suggested the two-step modulus-based matrix splitting iteration method and discussed its convergence, when the system matrix is an H_+-matrix and $A = M_1 - N_1 = M_2 - N_2$ are H-compatible splittings. Afterward, in 2014, Ke and Ma [24] established new convergence conditions. To improve the convergence conditions presented in [49], Zhang [47] assumed Ω satisfy $\Omega \geq D_A$ for the positive diagonal matrix Ω. Among the written articles in the field of the two-step modulus-based matrix splitting iteration method, we can mention [26, 47, 49, 54].

In recent years, the two-sweep modulus-based method has been studied as another effective module-based method for solving $\mathrm{LCP}(q,A)$. For the first time, in 2016, by using the identical equation, the two-sweep modulus-based matrix splitting iteration method was extended to $\mathrm{LCP}(q,A)$ [42]. This method can also yield a series of two-sweep modulus-based matrix splitting iteration methods, such as two-sweep modulus-based Jacobi, two-sweep modulus-based Gauss-Seidel, etc. By putting another diagonal matrix parameter to the fixed point iteration formula, a general two-sweep modulus-based matrix splitting iteration method for $\mathrm{LCP}(q,A)$ was established. Also, Peng [33] introduced a relaxation two-sweep modulus-based matrix splitting iteration method for $\mathrm{LCP}(q,A)$ based on choosing different pairs of relaxation parameters. In this article, we use a combination of two-sweep and two-step modulus-based matrix splitting iteration methods to introduce two classes of more efficient methods for solving $\mathrm{LCPs}(q,A)$. The convergence analysis of these methods is studied under suitable conditions. This paper consists of the following section.

Section 2 contains necessary items that we need throughout the article. Section 3 presents a two-step two-sweep modulus-based matrix splitting iteration method and a two-sweep modulus-based matrix splitting type II iteration method for $\mathrm{LCP}(q,A)$. Section 4 discusses the convergence analysis. Section 5 uses three numerical examples to illustrate that the presented methods are feasible. Finally, we present conclusions.

2. Preliminaries

In this section, some of the necessary definitions, notations, and lemmas are briefly introduced, see [1, 33, 36, 39, 41]. For $A = (a_{ij}) \in R^{n\times n}$, A is called nonnegative