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A Hybrid Scheme of Level Set and Diffuse Interface Methods for Simulating Multi-Phase Compressible Flows

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Abstract. We propose a hybrid scheme combing the level set method and the multi-component diffuse interface method to simulate complex multi-phase flows. The overall numerical scheme is based on a sharp interface framework where the level set method is adopted to capture the material interface, the Euler equation is used to describe a single-phase flow on one side of the interface and the six-equation diffuse interface model is applied to model the multi-phase mixture or gas-liquid cavitation on the other side. An exact Riemann solver, between the Euler equation and the six-equation model with highly nonlinear Mie-Grüneisen equations of state, is developed to predict the interfacial states and compute the phase interface flux. Several numerical examples, including shock tube problems, cavitation problems, air blast and underwater explosion applications are presented to validate the numerical scheme and the Riemann solver.

AMS subject classifications: 76L05, 76T30, 74H15, 68U20, 47E05

Key words: Multi-phase flow, sharp interface, diffuse interface, level set, six-equation model, exact Riemann solver, Mie-Grüneisen equation of state.

1. Introduction

Compressible multi-phase flows are ubiquitous in nature and engineering. Their applications span a wide range of areas including air blast, underwater explosions, combustion, high-speed aerodynamics and cavitation erosion. The numerical simulation of the compressible multi-phase flows are of great interest and importance.

In the application problems above, not only the densities and pressures vary largely, but also the constitutive relations differ distinctly across the material interface. So

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the material interface between distinct fluids is very important in the modelling of multi-phase flows. Algorithms for the treatment of interface are typically established based on how the numerical diffusion at the interface is eliminated. Typically, there are two dominant types of numerical methods for the simulation of multi-phase flow, the sharp interface method and the diffuse interface method. In the sharp interface method, the interface is assumed to be a sharp contact discontinuity and different fluids are immiscible. Several Eulerian approaches, such as volume of fluid (VOF) method [39, 49], level set method [52, 56], moment of fluid (MOF) method [2, 4, 16] and front-tracking method [23, 58] are used extensively to capture the interface. In the diffuse interface method, the physical models and governing equations are derived from phase field theory. The material interface is represented by a thin diffuse layer, and the flow properties such as density and viscosity change smoothly in the form of hyperbolic tangent functions. Many algorithms of diffuse interface method which have attained widespread applications in dealing with complex interface deformation problems [46, 60] can be broadly classified into four major types, the seven-equation model [6, 43], the six-equation model [40, 41, 47, 48], the five-equation model [3, 21, 31, 61] and the four-equation model [1, 29, 37].

The sharp interface method is more accurate in representing the shape of the interface, while it is more expensive due to the interface reconstruction and geometric advection processes within this method. The diffuse interface method makes no attempt to track the material interface, but instead treats the flow as a mixture of two phases with an average mixture density. It is able to treat all the possible physics of multiphase flow, including the dynamical phase creation and interface creation, cavitation evolution and collapse. The main drawback of the diffuse interface method is the excessive smearing of the fluid interfaces due to the numerical diffusion of the hyperbolic solver. To counteract this diffusion, a number of approaches are developed and can be classified into two groups. In the first group, a high-order method, e.g., the Weighted Essentially Non-Oscillatory (WENO) scheme is used. Methods in the second group use a series of correction terms, such as anti-diffusion, pseudo-time sharpening techniques [38, 53, 54], reconstruction-based interface sharpening approaches [11, 38, 57] along with the governing equations to sharpen the profiles of volume fraction and densities.

One critical issue for both the sharp interface method and diffuse interface method, is to determine the interfacial states, due to the great discrepancy of densities and equations of state across the material interface. The accurate prediction of the interfacial states can be used to stabilize numerical diffusion in diffuse interface method and to compute numerical flux and interface motion in sharp interface method. One common approach to determine the interfacial states is to solve a multi-phase Riemann problem containing the fundamental physical and mathematical properties of the governing equations. The solution of a Riemann problem depends not only on the initial states on each side of the interface, but also on the form of governing equations and equations of state. For the Euler equation with nonlinear equation of state, a variety of methods have been proposed to solve the corresponding Riemann problems. For ex-