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IMPROVED HARMONIC INCOMPATIBILITY REMOVAL FOR SUSCEPTIBILITY MAPPING VIA REDUCTION OF BASIS MISMATCH *

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Abstract

In quantitative susceptibility mapping (QSM), the background field removal is an essential data acquisition step because it has a significant effect on the restoration quality by generating a harmonic incompatibility in the measured local field data. Even though the sparsity based first generation harmonic incompatibility removal (1GHIRE) model has achieved the performance gain over the traditional approaches, the 1GHIRE model has to be further improved as there is a basis mismatch underlying in numerically solving Poisson's equation for the background removal. In this paper, we propose the second generation harmonic incompatibility removal (2GHIRE) model to reduce a basis mismatch, inspired by the balanced approach in the tight frame based image restoration. Experimental results shows the superiority of the proposed 2GHIRE model both in the restoration qualities and the computational efficiency.

Mathematics subject classification: 35R30, 42B20, 45E10, 65K10, 68U10, 90C90, 92C55.
Key words: Quantitative susceptibility mapping, Magnetic resonance imaging, Deconvolution, Partial differential equation, Harmonic incompatibility removal, (Tight) wavelet frames, Sparse approximation.

1. Introduction

Quantitative susceptibility mapping (QSM) [1] is a novel noninvasive imaging method which visualizes the magnetic susceptibility distribution of a human body from a given local field

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perturbation data measured from the magnetic resonance imaging (MRI) signal. The magnetic susceptibility χ is a physical property of a material which relates a magnetization $\mathbf{M} = (M_1, M_2, M_3)$ and a magnetic field $\mathbf{H} = (H_1, H_2, H_3)$ through $\mathbf{M} = \chi\mathbf{H}$ [2]. The physiological and/or pathological processes alter the scalar tissue magnetic susceptibilities [2], whose visualization is becoming reasonably robust and accurate for practical applications [3]. Consequently, QSM recently covers a various range of clinical applications such as demyelination, inflammation, and iron overload in multiple sclerosis [4], neurodegeneration and iron overload in Alzheimer's disease [5], Huntington's disease [6], changes in metabolic oxygen consumption [7], hemorrhage including microhemorrhage and blood degradation [8], bone mineralization [9], and drug delivery using magnetic nanocarriers [10].

QSM is based on the post processing the phase data of a complex gradient echo (GRE) signal because the magnetic susceptibility distribution in an MR scanner induces the *total field* which can be captured by the phase shifts in the GRE signal [11, 12]. The post processing consists of the four stages; the phase offset correction, the phase unwrapping, the background field removal, and the dipole inversion; see Fig. 1.1 for the brief overview of the process. Throughout this paper, we only focus on the background field removal to estimate the *local field* induced by the susceptibility in the region of interest (ROI) $\Omega \subseteq \mathbb{R}^3$ which occupies the water and brain tissues, and the dipole inversion to reconstruct and visualize the susceptibility distribution in the ROI Ω using the measured local field data. Interested readers may refer to e.g. [12] and references therein for more details on the other QSM stages such as the phase offset correction, the phase unwrapping, etc.

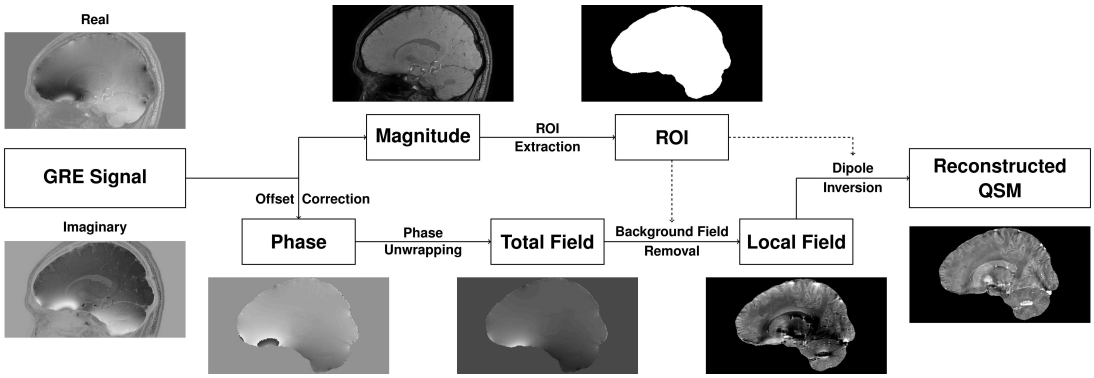

Fig. 1.1. Schematic diagram of QSM reconstruction process. Dashed line indicates that the extracted ROI is used for the background removal and/or the dipole inversion.

Given the local field f, the susceptibility reconstruction is based on solving the following convolution relation [13–15]:

$$f(\boldsymbol{x}) = \mathrm{pv} \int_\Omega d(\boldsymbol{x} - \boldsymbol{y})\chi(\boldsymbol{y})\mathrm{d}\boldsymbol{y}, \tag{1.1}$$

where pv denotes the principal value [16] of the singular integral with the kernel d:

$$d(\boldsymbol{x}) = \frac{2x_3^2 - x_1^2 - x_2^2}{4\pi|\boldsymbol{x}|^5}.$$