JOURNAL OF PARTIAL DIFFERENTIAL EQUATIONS
J. Part. Diff. Eq., Vol. **35**, No. 4, pp. 395-408

doi: 10.4208/jpde.v35.n4.7
December 2022

Boundedness for a Class of Operators on Weighted Morrey Space with RD-measure

CUI Xiaona[1], LU Yongjin[2,*] and LI Mengmeng[3]

[1] *Department of Mathematics and Information Science, Henan Normal University, Xinxiang 453007, China.*
[2] *Department of Mathematics and Statistics, Oakland University, Rochester, MI, 48309, USA.*
[3] *College of International Education, Henan Normal University, Xinxiang 453007, China.*

Received 25 May 2021; Accepted 19 April 2022

Abstract. In this paper, we study a class of sublinear operators and their commutators with a weighted BMO function. We first give the definition of a weighted Morrey space $L^{p,\kappa}_{\mu,\omega}(X)$ where X is an RD-measure and ω is the weight function. The weighted Morrey spaces arise from studying the local behavior of solutions to certain partial differential equations. We will show that the aforementioned class of operators and their communtators with a weighted BMO function are bounded in the weighted Morrey space $L^{p,\kappa}_{\mu,\omega}(X)$ provided that the weight function ω belongs to the $A_p(\mu)$-class and satisfies the reverse Hölder's condition.

AMS Subject Classifications: 35Q30, 35B40, 35B41, 76D03, 76D05

Chinese Library Classifications: O175.29

Key Words: Weighted Morrey space; RD-measure; commutator.

1 Introduction

In this paper, one subject is related to the functional spaces of homogeneous type including RD-spaces (or measure), which have been studied in recent decades by many researchers. For example, Han, Müller and Yang [1] developed an abstract theory of Triebel-Lizorkin and Besov spaces based on RD-spaces. In [2], the authors established a Littlewood-Paley theory of Hardy spaces $H^p(X)$ for $p\in(n/(n+1),1]$. The para-product

*Corresponding author. *Email addresses:* `cxn009@126.com` (X. N. Cui), `ylu2@oakland.edu` (Y. J. Lu), `hsdlmm@163.com.` (M. M. Li)

operators on RD-spaces were studied in [3] and [2]. Yang and Zhou [4] gave several equivalent characterizations of RD-spaces and provided the definition of the test functions on an RD-space firstly, which indicates that the Besov and Triebel-Lizorkin spaces on an RD-space are independent from the choice of the underlying distribution space. Then, in [5], the same authors studied the localized Hardy spaces related to admissible functions on RD-Spaces and their applications.

Another important ingredient of our current paper is the BMO (bounded mean oscillation) functions with weight ω (see Section 2), which was introduced in [6] first and then was generalized to some new weight-types in [7–10], such as the Bloom's BMO function in [7], BMO functions with Gaussian measure in [11]. The equivalence of various BMO spaces is shown in [9].

The classical Morrey space $L^{p,\lambda}$ was introduced by Morrey in [12] for the purpose of studying the local behavior of solutions for second order elliptic partial differential equations. Recently, Komori and Shirai [13] defined the weighted Morrey spaces $L^{p,\kappa}(w)$ and studied the boundedness of the classical singular integral operators and their commutators with the BMO functions on these spaces. Then, Wang [14] showed the boundedness of the commutator $[b,T]$ on the weighted Morrey spaces under some restriction on indexes p and κ, where b belongs to the weighted BMO or Lipschitz (with weight) space.

In this work, the RD-measure X that we consider is a space of functions of homogeneous type in the sense of Coifman and Weiss as defined in [15,16] and it also satisfies the additional reverse doubling property as defined in [2]. Our main goal of this paper is to show that a class of operators and their commutators with a weighted BMO function are bounded in $L^{p,\kappa}_{\mu,\omega}(X)$, where $L^{p,\kappa}_{\mu,\omega}(X)$ is a weighted Morrey space with the weight function ω belonging to the $A_p(\mu)$-class (which we will specify below).

Some of the major departures of our work from those available in the literature are: (1) By using an RD-measure, the weighted Morrey space that we define in this work generalizes the one defined in [17]; (2). The class of the operators that we consider commmute with a weighted BMO function rather than a BMO function as considered in [17].

In order to show our main result, the boundedness of the class of the operators and their commutators with a weighted BMO function in the weighted Morrey space, the doubling and reverse doubling property on the underlying RD-measure are critical. In addition, we have to also impose some key conditions on the weight function ω, which has to be from the $A_p(\mu)$-class and satisfies the reverse Hölder's condition. We will specify these conditions in Section 2 below.

The rest of the paper is outlined as below: in Section 2, we define some important functional spaces, including the weighted Morrey space $L^{p,\kappa}_{\mu,\omega}(X)$; in Section 3, we establish one of the main results: the boundedness of a class of sublinear operators in the weighted Morrey space $L^{p,\kappa}_{\mu,\omega}(X)$; in Section 4, we will then show the other main result: the boundedness of the commutator of the aforementioned class of sublinear operators with a weighted BMO function in the weighted Morrey space $L^{p,\kappa}_{\mu,\omega}(X)$.